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Aggregation in a Swarm of Non-Holonomic Agents Using Artificial Potentials and Sliding Mode Control

Veysel Gazi, M. İlter Köksal, and Barış Fidan

Abstract— **In this paper we consider the aggregation of swarms whose agents are moving in 2-dimensions with a non-holonomic agent dynamics. We approach this problem using artificial potentials and sliding mode control. The main contribution is the extension of the recent results in the literature based on a similar approach for simple integrator agent dynamics model to a significantly more realistic and more difficult setting with non-holonomic unicycle agent dynamics model. In particular, we design a continuous-time control scheme via a constructive analysis based on artificial potentials and sliding mode control techniques. The effectiveness of the proposed design for solving the swarm aggregation problem is demonstrated analytically as well as via a set of simulation results.**

I. INTRODUCTION

Aggregation (or gathering together) is a basic behavior exhibited by many swarms in nature, including simple bacteria colonies, flocks of birds, schools of fish, and herds of mammals. Such behavior of biological swarms is observed to be helpful in meeting various tasks such as avoiding predators, increasing the chance of finding food, etc. [1]. This can be explained by the relative appropriateness of an aggregated swarm structure to meet these tasks collaboratively as compared to a non-aggregated setting. Because of the same reason, aggregation is a desired behavior in engineering multi-agent dynamic systems as well. Moreover, many of the collective behaviors seen in biological swarms and some behaviors to be possible implemented in engineering multi-agent dynamic systems emerge in aggregated swarms. Therefore, studying the dynamics and properties of swarm aggregations is important in developing efficient cooperative multi-agent dynamic systems.

Aggregation in biological swarms were initially modelled and simulated by biologists [2], [3], [4], [5]. Inspired by these works, a recent series of studies [6], [7], [8], [9], [10], [11], [12], [13] has provided rigorous stability and convergence analysis of swarm aggregations based on artificial potential functions both with continuous-time and discrete-time formulations. Particularly, in [6], [7] a biologically inspired n-dimensional (where n is arbitrary) continuous time synchronous swarm model based on artificial potentials is considered and some results on cohesive swarm aggregation have been obtained. Similar results based on artificial potentials and virtual leaders have been independently obtained by Leonard and coworkers in [14], [15] for agents with point mass dynamics. The papers [10], [11], [12] focus on asynchronous swarm models with time delays for swarm aggregation in discrete-time settings.

In [13], which has more emphasis on design than analysis as opposed to the papers mentioned in the previous paragraph, a particular control strategy for aggregation in swarms has been developed based on artificial potential functions and sliding mode control, assuming simple integrator agent dynamics with model uncertainties and disturbances. The main contribution of this paper is the extension of the results in [13] to a significantly more realistic and more difficult setting with non-holonomic unicycle agent dynamics models, again using the tools of artificial potential functions and sliding mode control, but in a slightly different way than [13].

Artificial potential functions have been used extensively for robot navigation and control, see e.g. [16], [17]. There exist a number of more recent studies on applications of artificial potentials to multi-agent system coordination and cooperative control [18], [19]. There is also a relevant literature on formation control of autonomous vehicles [20], [21], [22], [23], [24], [25] as well as control and analysis of flocking behavior [26], [27], [28], [29], where artificial potential functions are used together with a number of other techniques including some graph theoretical and Lyapunov analysis based ones. Some of these works are based on point mass agent dynamics [18], [19], [24], [25], [26], [27], [28], while others use non-holonomic agent dynamics [20], [21], [22], [23], [29].

Sliding mode control [30], which is the main technique we use in our work in addition to artificial potential functions, is an important technique that has been used extensively in various areas including navigation of vehicles and mobile robots [31], [32], [33], [34], [35], [36]. The wide use of this technique for various tracking control problems is mainly because it is a robust technique which guarantees that the tracking is achieved in the existence of uncertainties and disturbances in the system dynamics.

In [31], [32], [33], sliding mode control is used for navigation of holonomic robots and obstacle avoidance in an environment modeled using harmonic potentials. In [34],

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[35], [36] the strategy is extended to navigation of robots with non-holonomic dynamics as well. The strategy in these works is based on forcing the motion of the robot along the gradient of the potential function representing the environment. In [13], a similar procedure is applied for implementation of a class of engineering aggregating swarm models composed of robots with fully actuated (holonomic) motion dynamics. As mentioned earlier, in this paper, we extend the study in [13] to a significantly more complex and realistic setting with non-holonomic unicycle agent dynamics models. In Section II, we present the mathematical swarm model with non-holonomic agent dynamics we are assuming in our work and define the aggregation problem we deal with. In Sections III and IV, we describe our control design for the aggregation problem defined in II based on artificial potential fields and sliding mode control. In Section V, we demonstrate the effectiveness and characteristics of our design via a simulation example. The paper is concluded with some final remarks in Section VI. An extended version of this article, where the foraging and formation control problems are considered (in addition to the aggregation problem considered here), can be found in [37].

II. Swarm Aggregation Problem with Non-Holonomic Agents

Consider a system of N non-holonomic mobile agents, e.g. robots, moving in $\mathbb{R}^2$ that are labelled as $A_1, \ldots, A_N$. Assume that each agent A_i ($i = 1, \ldots, N$) has the configuration depicted in Figure 1 and the equations of motion given by

$$\begin{array}{rcl} \dot{x}_i & = & v_i \cos(\theta_i), \\ \dot{y}_i & = & v_i \sin(\theta_i), \\ \dot{\theta}_i & = & w_i, \\ \dot{v}_i & = & \frac{1}{m_i} F_i, \\ \dot{w}_i & = & \frac{1}{I_i} \tau_i \end{array} \quad (1)$$

where x_i and y_i are the Cartesian coordinates, θ_i is the steering angle, v_i is the linear speed, and w_i is the angular speed of A_i. The quantities m_i and I_i are positive constants and represent the mass and the moment of inertia of the agent A_i, respectively. The control inputs for the agent A_i are the force input F_i and the torque input τ_i. Note that this model includes both kinematic and dynamic equations for each agent, i.e., it includes the (linear and angular) velocity dynamics in addition to the agent kinematics. This is equivalent to adding two integrators to the kinematic model.

In this article we are concerned with the aggregation problem for the agents with the dynamics given in (1). In other words, we would like to design the control inputs $u_{i1} = F_i$ and $u_{i2} = \tau_i$ such that the system of N agents with the non-holonomic dynamics given in (1) and with arbitrary initial positions aggregate (or gather) together.

Denoting the position of each agent A_i ($i = 1, \ldots, N$) by $p_i = [x_i, y_i]^\top$, we can formulate our control problem as follows.

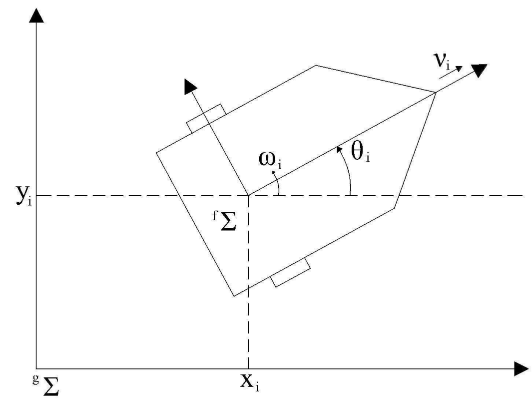

Fig. 1. The unicycle robot.

Problem 1: Design the control inputs $u_i = [u_{i1}, u_{i2}]^\top$ for each agent $A_i, i = 1, \ldots N$, such that for some $\epsilon > 0$ as $t \to \infty$ we have

$$p_i \to B_\epsilon(p_c) \quad (2)$$

where $p_c = \frac{1}{N}\sum_{i=1}^N p_i$ is the centroid of the swarm and $B_\epsilon(p_c) = \{p \in \mathbb{R}^2 | \|p - p_c\| \leq \epsilon\}$ is the disk with radius ϵ around the centroid p_c.

Note that this problem can be formulated also as

$$\lim_{t\to\infty} \|p_i - p_j\| \leq 2\epsilon$$

for all i and j. Here the size of the swarm (or the gathering area) ϵ is a design parameter that can be chosen by the system designer. Note that for the above problem definition it is assumed that the agents have point dimensions (although moving with non-holonomic constraints). If the agents have certain predefined dimensions/size, then the swarm size ϵ cannot be chosen arbitrarily small and should be consistent with the number of agents and the area each agent occupies.

III. Artificial Potential Functions

In our approach to Problem 1, we use artificial potential functions in order to construct attractive-repulsive relations among the agents. In other words, our design procedure is based on a potential function which is selected such that the corresponding potential field is attractive for agent pairs with large inter-agent distances (in order to result in aggregation) and repulsive for short inter-agent distances (in order to avoid collisions between the robots). In our work, we use a particular potential function of the form considered in [6], [8], [7].

In [6], [7] it was shown for a certain class of potential functions $J(p)$ that if the agents move in the space $\mathbb{R}^n$ based on

$$\dot{p}_i = -\nabla_{p_i} J(p), \quad (3)$$

where $J : \mathbb{R}^{nN} \to \mathbb{R}$ is the potential function, $p = [p_1^\top, \ldots, p_N^\top]^\top \in \mathbb{R}^{nN}$ is the lumped vector of the positions $p_i \in \mathbb{R}^n$ of the agents A_i ($i = 1, \ldots, N$), then aggregation in the form defined in Problem 1 will be achieved.[1] The potential functions considered in [6], [7] satisfy

$$\nabla_{p_i} J(p) = \sum_{j=1, j\neq i}^{N} g(p_i - p_j), \; i = 1, \ldots, N \quad (4)$$

where $g : \mathbb{R}^n \rightarrow \mathbb{R}^n$ are odd functions (called attraction/repulsion functions) that represent the attraction and repulsion relationship between the individuals. Moreover, it was assumed that for any $\bar{p} \in \mathbb{R}^n$, $g(\bar{p})$ satisfies

$$g(\bar{p}) = -\bar{p}[g_a(\|\bar{p}\|) - g_r(\|\bar{p}\|)],$$

where $g_a(\|\bar{p}\|)$ represents the attractive part which dominates on large distances and $g_r(\|\bar{p}\|)$ represents the repulsive part which dominates on short distances. One potential function which satisfies these assumptions and was used in [6], [8] is

$$J(p) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left[\frac{a}{2}\|p_i - p_j\|^2 + \frac{bc}{2} \exp\left(-\frac{\|p_i - p_j\|^2}{c} \right) \right], \quad (5)$$

for which (4) is satisfied with

$$g(p_i - p_j) = -(p_i - p_j) \left[a - b \exp\left(-\frac{\|p_i - p_j\|^2}{c} \right) \right] \quad (6)$$

where a, b and c are positive scalars that need to be chosen appropriately. In particular, for the potential function in (5) the size of the gathering region in Problem 1 is given by

$$\epsilon = \frac{b}{a}\sqrt{\frac{c}{2}} \exp\left(-\frac{1}{2} \right).$$

Note also that the above value of ϵ is a very conservative bound obtained as a result of a Lyapunov analysis and in reality the actual swarm size is much smaller than it. Therefore, beside the value of ϵ another parameter that may give information about the size of the swarm could be the distances at which the attraction and repulsion between two individuals balance. For the potential function in (5) this happens at the distance

$$\delta = \sqrt{c \ln\left(\frac{b}{a} \right)}.$$

In this article, we also use the potential function (5). Note that in our case $p = [p_1^\top, \ldots, p_N^\top]^\top \in \mathbb{R}^{2N}$ and $p_i = [x_i, y_i]^\top \in \mathbb{R}^2$ for $i = 1, \ldots, N$.

IV. Sliding Mode Control for Swarm Aggregation

As mentioned in Section I, sliding mode control is a widely used technique in various application areas, mainly because of its suppressive and robust characteristics against the uncertainties and the disturbances in the system dynamics. The shortcomings (of the raw form of the sliding mode control scheme) on the other hand are the so-called chattering effect and possible generation of high-magnitude control signals. Note that these shortcomings may possibly be avoided or relaxed via integration and some filtering techniques.

In sliding mode control, a switching controller with high enough gain is applied to suppress the effects of modelling uncertainties and disturbances, and the agent dynamics are forced to move along a stabilizing manifold, which is also called a *sliding manifold*. The value of the gain is computed using the known bounds on the uncertainties and disturbances.

In this section, we design a sliding mode control scheme to solve Problem 1 via forcing the motion of each individual agent along the negative gradient of the potential $J(p)$ in (5), i.e. forcing each agent to obey equation (3) where J is as defined in (5). This will lead to recovering the aggregation behavior (of the single-integrator dynamics) obtained in [6] with non-holonomic agent dynamics.

Let

$$-\nabla_{p_i} J(p) = \begin{bmatrix} -J_{x_i}(p) \\ -J_{y_i}(p) \end{bmatrix}$$

denote the gradient of the potential at p_i. In order to achieve satisfaction of (3) we need

$$-\nabla_{p_i} J(p) = \begin{bmatrix} -J_{x_i}(p) \\ -J_{y_i}(p) \end{bmatrix} = \begin{bmatrix} v_i \cos\theta_i \\ v_i \sin\theta_i \end{bmatrix} \quad (7)$$

i.e.

$$v_i = \|\nabla_{p_i} J(p)\|, \quad \theta_i = \arctan\left(\frac{J_{y_i}(p)}{J_{x_i}(p)} \right) (\mathrm{mod} 180^\circ) \quad (8)$$

Note that since the inputs in the agent model (1) are $u_{i1} = F_i$ and $u_{i2} = \tau_i$, i.e. v_i and θ_i cannot be applied directly, the terms

$$v_{id} \triangleq \|\nabla_{p_i} J(p)\|, \quad \theta_{id} \triangleq \arctan\left(\frac{J_{y_i}(p)}{J_{x_i}(p)} \right) (\mathrm{mod} 180^\circ) \quad (9)$$

need to be considered as desired set-point values for v_i and θ_i, respectively.

Our objective is to force the motion of the agents such that the differences $(v_i - v_{id})$ and $(\theta_i - \theta_{id})$ converge to zero. With this objective in mind, similar to [34], [35], [36], let us define two sliding surfaces [30], one for the translational speed v_i and one for the orientation θ_i. Defining

$$s_{v_i} = v_i - v_{id} \quad (10)$$

$$s_{\theta_i} = c(\dot{\theta}_i - \dot{\theta}_{id}) + (\theta_i - \theta_{id}), \quad (11)$$

where $c > 0$ is a positive constant, the corresponding sliding surfaces are the surfaces at which $s_{v_i} = 0$ and $s_{\theta_i} = 0$. With these definitions, our objective becomes to design the control inputs u_{i1} and u_{i2} so that $s_{v_i} \rightarrow 0$ and $s_{\theta_i} \rightarrow 0$ asymptotically, since if they are achieved we will have $v_i \rightarrow v_{id}$ and $\theta_i \rightarrow \theta_{id}$. Note here that the existence of the additional term $c(\dot{\theta}_i - \dot{\theta}_{id})$ in (11) is because of the double integrator relationship between θ_i and the applicable input $u_{i2} = \tau_i$ as opposed to the single integrator relationship between v_i and $u_{i1} = F_i$.

[1] Note that since we have $n = 2$ for the the model in (1), i.e., $p_i \in \mathbb{R}^2$, the corresponding potential function will satisfy $J : \mathbb{R}^{2N} \rightarrow \mathbb{R}$ for this case.

It is well known from the sliding mode control theory that if we have the reaching conditions

$$s_{v_i}\dot{s}_{v_i} \le -\varepsilon_1 |s_{v_i}| \quad (12)$$

$$s_{\theta_i}\dot{s}_{\theta_i} \le -\varepsilon_2 |s_{\theta_i}| \quad (13)$$

satisfied for some constants $\varepsilon_1, \varepsilon_2 > 0$, then $s_{v_i} = 0$ and $s_{\theta_i} = 0$ will be achieved in finite time.

Now let us assume that $|\dot{v}_{id}| \le \alpha(p)$ for some known $\alpha(p) > 0$. The properties of such $\alpha(p)$ depend on the properties of the potential function, which is chosen by the designer. In other words, one can choose the potential function such that such $\alpha(p)$ exists. For example, for the potential function in (5) one can calculate (see [37]) $\alpha(p)$ as

$$\alpha(p) = 2\bar{\alpha}(p) \max_{i\in\{1,\ldots,N\}} \left(\sum_{j=1, j\ne i}^{N} \|G(p_i - p_j)\| \right),$$

where

$$\bar{\alpha}(p) = \max_{k\in\{1,\ldots,N\}} \left(\|\nabla_{p_k} J(p)\| + s_{v_k}(0)\right).$$

and

$$G(p_i - p_j) = aI + b\exp\left(-\frac{\|p_i - p_j\|^2}{c}\right)\left(\frac{2}{c}(p_i - p_j)(p_i - p_j)^\top - I\right).$$

In order to achieve the satisfaction of (12) we choose the first control input $u_{i1} = F_i$ as

$$u_{i1} = -A_{i1}\operatorname{sgn}(s_{v_i}) \quad (14)$$

using which the time derivative of s_{v_i} becomes

$$\dot{s}_{v_i} = -\frac{A_{i1}}{m_i}\operatorname{sgn}(s_{v_i}) - \dot{v}_{id}$$

and we have

$$\begin{aligned} s_{v_i}\dot{s}_{v_i} &= s_{v_i}\left(-\frac{A_{i1}}{m_i}\operatorname{sgn}(s_{v_i}) - \dot{v}_{id}\right) \\ &= -\frac{A_{i1}}{m_i}|s_{v_i}| - s_{v_i}\dot{v}_{id} \\ &\le -\left(\frac{A_{i1}}{m_i} - \alpha(p)\right)|s_{v_i}| \end{aligned} \quad (15)$$

Then by choosing A_{i1} according to

$$A_{i1} \ge m_i(\alpha(p) + \varepsilon_1) \quad (16)$$

one guarantees that (12) is satisfied and sliding mode occurs (i.e., $s_{v_i} = 0$ is satisfied) in finite time.

Similarly, for the second sliding surface choosing the control input as

$$u_{i2} = -A_{i2}\operatorname{sgn}(s_{\theta_i}) \quad (17)$$

the time derivative of s_{θ_i} becomes

$$\dot{s_{\theta_i}} = -c\frac{A_{i2}}{I_i}\operatorname{sgn}(s_{\theta_i}) - c\ddot{\theta}_{id} + \omega_i - \dot{\theta}_{id} \quad (18)$$

and we have

$$\begin{aligned} s_{\theta_i}\dot{s_{\theta_i}} &= s_{\theta_i}\left(-\tfrac{cA_{i2}}{I_i}\operatorname{sgn}(s_{\theta_i}) - c\ddot{\theta}_{id} + \omega_i - \dot{\theta}_{id}\right) \\ &\le -\left(\tfrac{cA_{i2}}{I_i} - c|\ddot{\theta}_{id}| - |\dot{\theta}_{id}| - |\omega_i|\right)|s_{\theta_i}| \end{aligned} \quad (19)$$

By choosing A_{i2} as

$$A_{i2} \ge \frac{I_i}{c}\left(c|\ddot{\theta}_{id}| + |\dot{\theta}_{id}| + |\omega_i| + \varepsilon_2\right) \quad (20)$$

one can guarantee that (13) is satisfied and the second sliding surface $s_{\theta_i} = 0$ will as well be reached in finite time (11).

In order to be able to compute the value of s_{θ_i} one needs the time derivative of θ_{id}. Taking its derivative with respect to time we obtain

$$\begin{aligned} \dot{\theta}_{id} &= \frac{\frac{d}{dt}\left(\frac{J_{y_i}}{J_{x_i}}\right)}{1 + \left(\frac{J_{y_i}}{J_{x_i}}\right)^2} \\ &= \frac{\frac{d}{dt}(J_{y_i})\cdot J_{x_i} - \frac{d}{dt}(J_{x_i})\cdot J_{y_i}}{(J_{x_i})^2\left(1 + \left(\frac{J_{y_i}}{J_{x_i}}\right)^2\right)} \\ &= \frac{\frac{d}{dt}(J_{y_i})\cdot J_{x_i} - \frac{d}{dt}(J_{x_i})\cdot J_{y_i}}{(J_{x_i})^2 + (J_{y_i})^2} \end{aligned} \quad (21)$$

For the potential function in (5) we have

$$\begin{aligned} \tfrac{d}{dt}(J_{x_i}) = \textstyle\sum_{j=1, j\ne i}^{N} \Bigg[& -\left[a - b\left(1 - \tfrac{2(x_i - x_j)^2}{c}\right)\exp\left(-\tfrac{\|p_i - p_j\|^2}{c}\right)\right](\dot{x}_i - \dot{x}_j) \\ & + \left[b\tfrac{2(x_i - x_j)(y_i - y_j)}{c}\exp\left(-\tfrac{\|p_i - p_j\|^2}{c}\right)\right](\dot{y}_i - \dot{y}_j)\Bigg] \end{aligned}$$

and

$$\begin{aligned} \tfrac{d}{dt}(J_{y_i}) = \textstyle\sum_{j=1, j\ne i}^{N} \Bigg[& -\left[a - b\left(1 - \tfrac{2(y_i - y_j)^2}{c}\right)\exp\left(-\tfrac{\|p_i - p_j\|^2}{c}\right)\right](\dot{y}_i - \dot{y}_j) \\ & + \left[b\tfrac{2(x_i - x_j)(y_i - y_j)}{c}\exp\left(-\tfrac{\|p_i - p_j\|^2}{c}\right)\right](\dot{x}_i - \dot{x}_j)\Bigg]. \end{aligned}$$

One drawback here is that to implement the control algorithm for agent A_i one needs not only the position but also the velocity of its neighbors (which are all the other agents in the particular setting here - but this is not necessarily required to be the case in general).

We would like to also emphasize that although not explicitly considered here the procedure based on the sliding mode control techniques discussed above will guarantee proper behavior even in the presence of uncertainties in the mass m_i and the inertia I_i of the robots and disturbances and additive disturbances/uncertainties to the linear and angular speed dynamics which constitute very realistic assumptions.

Once the sliding mode occurs on all the surfaces (which happens in finite time) and the equation in (3) is also satisfied, based on the results in [6] we know that Problem 1 will be solved. One issue to note, however, is that after occurrence of sliding mode we reach $v_i = v_{id}$ but not necessarily $\theta_i = \theta_{id}$. In fact, after occurrence of sliding mode we have

$\theta_i \to \theta_{id}$ exponentially fast and the speed of convergence depends on the slope of the sliding surface $-\frac{1}{c}$. Therefore, one needs to choose c as small as possible in order to achieve faster convergence and avoid any instabilities. Note also that decreasing the parameter c will require increasing the controller gain A_{i2}.

V. SIMULATION RESULTS

In this section we present simulation results to test the effectiveness of the method used. In order to test the method we ran the program many times with various simulation parameters. By doing so we also observed the effects of the simulation parameters.

The potential function that we used in the simulations is the function with linear attraction and exponential repulsion given in (5). We tested the method for various function parameters a, b, and c, which control the swarm size (as well as prevent collisions). Moreover, we performed simulations for different number of agents in the swarm.

The sliding mode control method uses the sgn function to calculate the control inputs u_{i1} and u_{i2}. Although this works very well in theory, in practice it may result in high frequency chattering and numerical problems. There are various methods for smoothing the sliding mode control input to reduce the chattering. The analysis of such techniques is out of the scope of this article. Still however, we used the $\tanh(\gamma y)$ function instead of the the $\operatorname{sgn}(y)$ function in the simulations, where $\gamma > 0$ is a smoothness parameter.

The values of the control gains A_{i1} and A_{i2} are also very important and they need to be chosen "hight enough" in order for the procedure to work properly. They also affect the reaching time to the respective sliding surfaces. The constant parameter $c > 0$ used in the derivation of the sliding surface (11) which is used for orientation control determines the slope of the sliding line ($-\frac{1}{c}$ is the slope) that controls the speed of the exponential decay of $(\theta_i - \theta_{id})$ to zero after the sliding surface is reached. It provides also a smoother rotation for the agent.

Since the simulations obtained for different parameters and agent numbers are in principle the same (do not differ qualitatively) here we show only the ones for $N = 10$ and potential function parameters $a = 0.01$, $b = 20$, and $c = 1$. The control input gains are chosen as $A_{i1} = A_{i2} = 10$. The slope of the orientation sliding line/surface is chosen as $c = 0.5$. The smoothness/sharpness parameter for the $\tanh$ function for both of the control inputs are chosen as $\gamma_{i1} = \gamma_{i2} = 10$. Without loss of generality the mass and the inertia of all the agents are chosen equal and in particular $m_i = 1$ and $I_i = 1$ for all $i = 1, \ldots, N$. The simulation results show that the agents aggregate as predicted by the theory.

Figure 2 shows the motion (the trajectories) for random initial positions and orientations. The agents are plotted as polygons so that their orientations are explicitly shown. It is observed that the agents aggregate quickly and after aggregation they start to reorient themselves since at the aggregate the variation of the time-varying potential function (which is due to the motion of the other agents in the group) is higher.

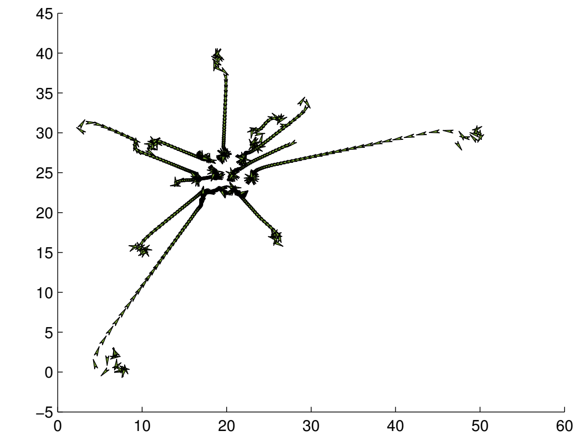

Fig. 2. Trajectories of the agents of a 10-agent swarm during the aggregation process.

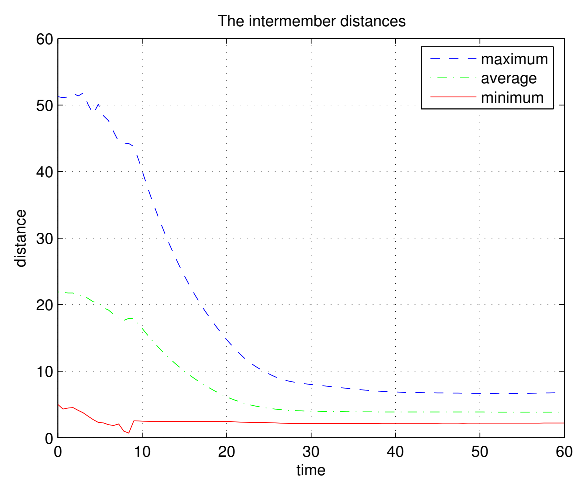

Fig. 3. Inter-agent distances during the aggregation process.

In Figure 3 we see the inter-member distances between robots. The curves specify the maximum, minimum and average distances between the members of the swarm. The distance decreases exponentially as expected and they converge to constant values similar to the results obtained before in [6]. For the above values of the parameters a, b and c the distance at which the attraction and repulsion between two individuals balance is $\delta = 2.7570$.

In [6] it was shown that the centroid of the swarm will be stationary for all time. Here this is guaranteed to be the case once sliding mode occurs on all surfaces and the orientations and the speeds of all the agents converge to the desired values. Therefore, although initially the center may not be stationary, after a while it must become stationary. Figure 4 shows the plot of the center movement. Keeping in mind that we are working in 2-dimensional space, this plot shows

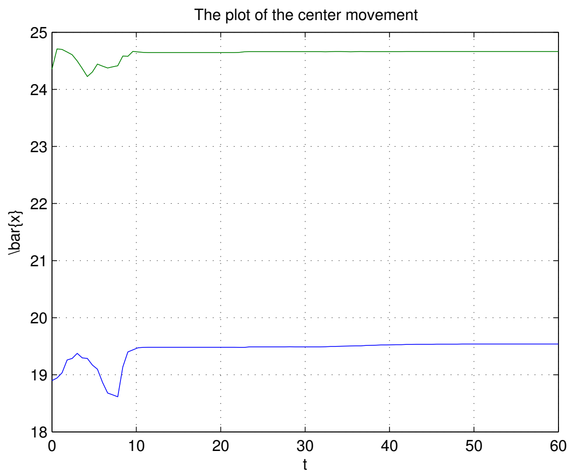

Fig. 4. The x and y coordinates of the swarm center during the aggregation process.

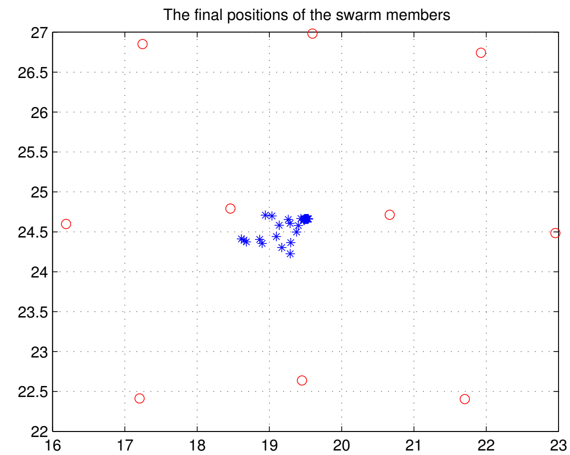

Fig. 6. Final positions of the agents whose trajectories are given in Figure 2.

the center movement in x-axis and y-axis. As expected after a while the location of the center converges to a constant position and stays there during the rest of the simulation.

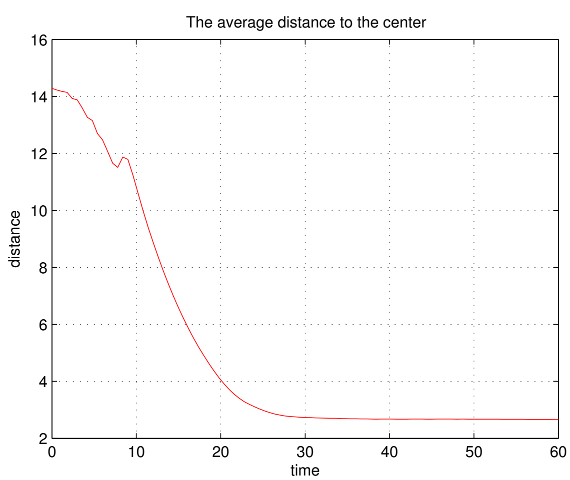

Fig. 5. Average agent distance to the swarm center during the aggregation process.

Figure 5 is the plot of the average distance of the swarm members to the center of the swarm. The value decreases exponentially during the simulation. It is stable and smooth.

Figure 6 shows the final positions of the swarm members (shown as circles) and the center location movement (shown as stars).

An interesting observation here is that at their final positions the swarm members are distributed in almost a grid-like arrangement. Also one should note that the distances between final positions of swarm members change for different values of attraction and repulsion parameters. For example, decreasing the repulsion parameter b results in decrease in the inter-agent distances at the final positions.

VI. Concluding Remarks

In this article we developed a strategy for aggregation of a swarm of non-holonomic agents based on artificial potential functions and the sliding mode control technique. The method is based on forcing the motion of the agents along the gradient field of the potential function generated based on the inter-individual distance requirements in the swarm aggregate. The method can be extended to the formation control and foraging problems as well [37]. Possible future research directions could include extending the setting to tracking desired trajectories or moving targets. In particular, the agents may be required to keep a certain geometrical formation while the centroid p_c of the swarm is required to track a certain trajectory. Other issues that could be considered are the performance and modifications of the algorithm under inter-agent distance measurement errors.

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