Tracking a Maneuvering Target with a Swarm of Non-holonomic Agents Using Artificial Potentials and Sliding Mode Control

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Abstract

In this article we consider tracking a maneuvering target with a swarm of non-holonomic agents. The target and the agents move in 2-dimensional space. The tracking agents are required to capture and enclose the target while moving as a geometric formation. In order to achieve this cooperative task, we design a decentralized continuous-time control scheme via constructive analysis using artificial potentials and sliding mode control techniques. The main contribution of this paper is the extension of recent results for the same task assuming simple integrator and fully actuated agent dynamics models to a significantly more realistic and more difficult setting with non-holonomic unicycle agent dynamics model. The effectiveness of the proposed control scheme is established analytically and demonstrated via a set of simulation results.

I. Introduction

In recent years, many research studies have focused on adapting biological principles to control systems in order to develop systems that are inspired from biology (see, e.g., [1]–[6]). These works range from modelling the behaviors of biological swarms (bacteria colonies, flocks of birds, schools of fish and herds of mammals) to mimicking the biological behaviors in order to build intelligent algorithms that solve multi-agent engineering system problems. The motivation behind adapting such biological systems to engineering problems is that these systems are very well suited to their environments.

A particular class of the studies mentioned above is on distributed coordination and control of multiple autonomous agents [1]–[6]. A group of very simple agents can cooperatively perform tasks that are too complex for a single one. Also, if one of the members of the group fails, others can re-coordinate themselves to complete the task. Working this way, agents increase the robustness of the system and probability of success. These cooperative systems are implemented in real-life in the form of autonomous robot teams, mobile sensor networks, groups of manned or unmanned aerial, ground, space or underwater vehicles, etc. [6]–[11].

In this paper, we consider a particular distributed coordination and control problem: Coordinated tracking of a maneuvering target with a swarm of autonomous agents. The particular task is development of a distributed control strategy for multi-agent systems (swarms) to enclose a moving target while moving in a specific geometric formation in $\mathbb{R}^2$. This problem has been investigated in [3] for two assumption cases, in one of which the agents are assumed to have simple integrator dynamics

$$\dot{p}_i = u_i \tag{1}$$

and in the other they are assumed to have fully-actuated dynamics

$$M(p_i)\ddot{p}_i + f_i(p_i, \dot{p}_i) = u_i \tag{2}$$

where p_i and u_i denote the position (in $\mathbb{R}^2$) and the control signal of the i'th agent in the swarm, respectively. Also, $M(p_i)$ and $f_i(p_i, \dot{p}_i)$ denote the relevant mass (inertia) matrix and the cumulative disturbance for agent i, respectively.

Both of the models (1) and (2) significantly simplify the actual agent dynamics that would be encountered in practice. In this paper, in place of (1) and (2), we assume a significantly more realistic and more difficult setting with non-holonomic unicycle agent dynamics model, which will be described in detail in Section II. The main contribution of the paper is the extension of the results in [3] for the simple integrator and fully actuated agent dynamics cases to the non-holonomic unicycle agent dynamics setting. Note here that this extension is not a straightforward one as the control design approach used in [3] cannot be directly applied to the non-holonomic unicycle agent dynamics model.

The coordinated tracking task stated above is actually consisting of two-subtasks for the swarm: Tracking a moving target and maintaining the geometric swarm formation (shape). In order to perform these two sub-tasks simultaneously, we consider a distributed control strategy based on artificial potential functions and sliding mode techniques [12], [13].

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In the literature, there exist applications of artificial potentials and/or sliding mode control to a number of target tracking and/or formation control problems (see, e.g., [2], [3], [6], [14]–[23]). However none of these applications (other than [3]) is in the context of the particular tracking task mentioned above.

Since the control design approach in [3] is not directly applicable to the non-holonomic unicycle agent dynamics setting of this paper, as a basis, we use the artificial potential and sliding mode control based approach of [24], which is used there to design distributed control schemes for aggregation, foraging, and formation acquisition/maintenance of swarms with non-holonomic agents.

The paper is organized as follows. In Section II, the assumed non-holonomic agent model is introduced and the particular coordinated tracking problem is defined. In Section III, the control design procedure for the solution of the tracking problem is explained. In Section IV, some simulation results are presented. Finally, the paper is concluded with some final comments in Section V.

II. Swarm Tracking Problem with Unicycle Agent Dynamics

Consider a system of N non-holonomic mobile agents, e.g. robots, moving in $\mathbb{R}^2$ that are labelled as $A_1, \ldots, A_N$. Assume that each agent A_i ($i = 1, \ldots, N$) has the configuration depicted in Figure 1 and the equations of motion given by

$$\begin{array}{rcl} \dot{x}_i &=& v_i \cos(\theta_i), \\ \dot{y}_i &=& v_i \sin(\theta_i), \\ \dot{\theta}_i &=& w_i, \\ \dot{v}_i &=& \frac{1}{m_i}\left[F_i + f_{v_i}\right], \\ \dot{w}_i &=& \frac{1}{I_i}\left[\tau_i + f_{w_i}\right] \end{array} \tag{3}$$

where x_i and y_i are the Cartesian coordinates, θ_i is the steering angle, v_i is the linear speed, and w_i is the angular speed of A_i. The quantities m_i and I_i are positive constants and represent the mass and the moment of inertia of the agent A_i, respectively. The control inputs for the agent A_i are the force input F_i and the torque input τ_i. The functions f_{v_i} and f_{w_i} represent additive disturbances for each agent A_i. The disturbances are bounded such that $|f_{v_i}| < f_v^+$ and $|f_{w_i}| < f_w^+$ for known bounds f_v^+ and f_w^+. The exact values of mass m_i and inertia I_i are unknown for each agent A_i ($i = 1, \ldots, N$), but bounds $0 < \underline{M} < m_i < \overline{M}$ and $0 < \underline{I} < I_i < \overline{I}$ are known. Note that this model includes both kinematic and dynamic equations for each agent, i.e., it includes the (linear and angular) velocity dynamics in addition to the agent kinematics. This is equivalent to adding two separate integrators to the kinematic model.

Remark 1: In this article, all the angles including θ_i are assumed to take values from the $[0°, 360°)$ interval. Because of this, all addition operations on the angles will be (mod $360°$). For example $\theta_1 - \theta_2$ means $(\theta_1 - \theta_2)(\text{mod } 360°)$. Similarly $\dot{\theta}_i(t)$ will be defined as

$$\dot{\theta}_i(t) = \lim_{\Delta t \to 0} \frac{(\theta_i(t) - \theta_i(t - \Delta t))(\text{mod } 360°)}{\Delta t}.$$

In the case above all the angles are on a circle so there won't be any discontinuity.

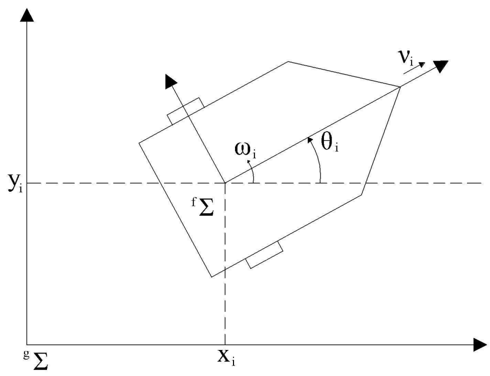

Fig. 1. Illustration of agent A_i with the non-holonomic unicycle dynamics.

Our aim in this paper is to track a maneuvering target with a swarm of non-holonomic robots with the dynamics given in (3). In other words, we would like to design the control inputs $u_{i1} = F_i$ and $u_{i2} = \tau_i$ such that the system of N agents follow the escaping target in a predefined geometrical shape (or formation) and enclose it. Let $p_i(t) = [x_i(t), y_i(t)]^\top$ denote the position of agent A_i for $i = 1, \ldots N$ and $p_T(t) = [x_T(t), y_T(t)]^\top$ denote the position of the target, at a certain time instant t. We can formulate the problem as follows.

Problem 1: (Swarm Tracking Problem) Consider a swarm of N agents A_i, $i \in \{1, \ldots, N\}$, where each agent A_i of the swarm has motion dynamics given by (3). Assume that each agent A_i, $i \in \{1, \ldots, N\}$, can sense the position $p_j(t)$ of any agent A_j in the swarm including itself and the position $p_T(t)$ of the target at any time instant t. Assume also that the velocity and the accelaration of the target are not known; however, they are bounded $\|\dot{p}_T(t)\| \leq \beta_{T_v}$, $\|\ddot{p}_T(t)\| \leq \beta_{T_a}$ for known bounds β_{T_v} and β_{T_a} respectively. Given a set of desired inter-agent distances $\{d_{ij} | i, j \in \{1, \ldots, N\}, i \neq j\}$, where d_{ij} denotes the desired distance between agents A_i and A_j, design the control inputs

$$u_i = [u_{i1}, u_{i2}]^T = [F_i, \tau_i]^T$$

for each agent A_i, $i = 1, \ldots, N$, such that both the following are satisfied.

$$p_T(t) \rightarrow \text{conv}\{p_1(t), \ldots, p_N(t)\} \quad \text{as} \quad t \rightarrow \infty \tag{4}$$

$$\left| \lim_{t \rightarrow \infty} \|p_i(t) - p_j(t)\| - d_{ij} \right| \leq \epsilon, \ \forall i, j \in \{1, \ldots, N\}, i \neq j \tag{5}$$

where conv$\{p_1, \ldots, p_N\}$ denotes the convex hull of $p_1, \ldots, p_N$ and $\epsilon > 0$ is a small design constant.

In Problem 1, equation (5) formulates the formation acquisition/maintenance subtask whereas equation (4) formulates the subtask of tracking/enclosing the moving target. We approach Problem 1 using artificial potentials and sliding mode control. The artificial potential function to be used will be a composition of two functions (or two function sub-compositions), one corresponding to the target tracking requirements (4), and one to the formation control requirement (5).

III. Control Design

A. Artificial Potential Functions

In our approach to Problem 1, we use artificial potential functions in order to construct attractive-repulsive relations among the agents and between agents and the target. The potential function must satisfy both the tracking and the formation specifications. In other words, our design procedure is based on a potential function that is composed of two parts - a formation control part and a tracking part. In this work, we use a particular class of potential functions of the form considered in [3], [22], [24]–[26]:

$$J(p_1, ..., p_N, p_T) = J_T(p_1, .., p_N, p_T) + J_F(p_1, .., p_N), \tag{6}$$
$$J_T(p_1, \ldots, p_N, p_T) = w_T \sum_{i=1}^{N} J_{iT}(\|p_i - p_T\|),$$
$$J_F(p_1, \ldots, p_N) = w_F \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} J_{ij}(\|p_i - p_j\|).$$

where

$$J_{iT}(\|p_i - p_T\|) = \frac{1}{4}\|p_i - p_T\|^4. \tag{7}$$

$$J_{ij}(\|p_i - p_j\|) = \frac{1}{2}\left[a_{ij}\|p_i - p_j\|^2 + b_{ij} c_{ij} e^{-\frac{\|p_i - p_j\|^2}{c_{ij}}}\right]. \tag{8}$$

Here $J_{iT}(\|p_i - p_T\|)$ is the potential between agent A_i and the target, and $J_{ij}(\|p_i - p_j\|)$ is the potential between agent A_i and agent A_j. The constants w_T and w_F denote the weights (which quantify the relative importance) of the tracking and formation control parts respectively. Note that the potential between the agents and that between the agents and the target are functions of the corresponding inter-distances. With such a potential function, every agent is forced to track the target while maintaining the formation.

An issue to note about the formation part of the potential function is that in [22], [25] it was shown for a certain class of potential functions $J_F(p_1, .., p_N)$ that if the agents move in the space $\mathbb{R}^2$ (our case) based on

$$\dot{p}_i = -\nabla_{p_i} J_F(p_1, .., p_N), \tag{9}$$

and if the artificial potential function is chosen with pair dependent inter-agent distances as in (8) the inter-agent attraction-repulsion forces balance at

$$\delta_{ij} \triangleq \sqrt{c_{ij} \ln\left(\frac{b_{ij}}{a_{ij}}\right)} = d_{ij}. \tag{10}$$

This implies that, as in [24], the potential function will have its global minimum (not necessarily unique) at the desired geometrical formation and provided that the initial agent configuration is "close enough" to the desired formation it will be achieved. This is basically due to the local minima problem present in the potential function based approaches.

Another issue to note is about the tracking part of the potential function: $J_{iT}(\|p_i - p_T\|)$ in (7) satisfies

$$\nabla_y J_{iT}(\|y\|) = y h_{iT}(\|y\|) \tag{11}$$

where

$$h_{iT}(\|y\|) \triangleq \|y\|^2. \tag{12}$$

Here $h_{iT}(\|y\|)$ is always greater than zero for all y except for $\|y\| = 0$. If we achieve $\nabla_{p_i} J(p_1, \ldots, p_N, p_T) = 0$ for all i, using the summation $\sum_{i=1}^{N} \nabla_{p_i} J(p_1, \ldots, p_N, p_T) = 0$ and from the reciprocity of the attraction/repulsion forces between the agents in (8) (which implies that $\sum_{i=1}^{N} \nabla_{p_i} J_F(p_1, \ldots, p_N, p_T) = 0$) one can show that [3]

$$\sum_{i=1}^{N} \nabla_{p_i} J_{iT}(\|p_i - p_T\|) = \sum_{i=1}^{N} (p_i - p_T) h_{iT}(\|p_i - p_T\|) = 0$$

is achieved. Rearranging this equation we obtain

$$\sum_{i=1}^{N} p_i h_{iT}(\|p_i - p_T\|) = p_T \sum_{i=1}^{N} h_{iT}(\|p_i - p_T\|).$$

Given the fact that $h_{iT}(\|p_i - p_T\|) \geq 0$ for all p_i and p_T and that $h_{iT}(\|p_i - p_T\|) = 0$ only when $p_i = p_T$ together with the fact that due to the inter-agent repulsion forces in (8) the agents cannot simultaneously occupy the same position with the target and always there is at least one agent A_i such that $p_i \neq p_T$ (implying that for that agent A_i we have $h_{iT}(\|p_i - p_T\|) > 0$) we have the inequality $\sum_{i=1}^{N} h_{iT}(\|p_i - p_T\|) \neq 0$ always satisfied. Then, rearranging the above equation we obtain

$$p_T = \frac{\sum_{i=1}^{N} p_i h_{iT}(\|p_i - p_T\|)}{\sum_{i=1}^{N} h_{iT}(\|p_i - p_T\|)}.$$

Defining

$$\eta_i \triangleq \frac{h_{iT}(\|p_i - p_T\|)}{\sum_{i=1}^{N} h_{iT}(\|p_i - p_T\|)}$$

for $i = 1, \ldots, N$ the equation becomes

$$p_T = \sum_{i=1}^{N} \eta_i p_i. \tag{13}$$

By definition we have $\sum_{i=1}^{N} \eta_i = 1$. Furthermore, for any $i \in \{1, \ldots, N\}$, since $h_{iT}(\|y\|) \geq 0$, we have $0 \leq \eta_i \leq 1$. This implies that as $t \to \infty$, if for all i $\nabla_{p_i} J(p_1, \ldots, p_N, p_T) \to 0$ is achieved, then $p_T \to \text{conv}\{p_1, \ldots, p_N\}$ is achieved as well. In other words, if as $t \to \infty$ the equality $\nabla_{p_i} J(p_1, \ldots, p_N, p_T) = 0$ is satisfied for all i, then condition (4) in Problem 1 is also satisfied.

B. Sliding Mode Control Design

Sliding mode control is a widely used technique in various application areas, including multi-agent system coordination and control as mentioned in Section I. This is mainly because of its suppressive and robust characteristics against the uncertainties and the disturbances in the system dynamics. The shortcomings (of the raw form of the sliding mode control scheme), on the other hand, are the so-called chattering effect and possible generation of high-magnitude control signals [12], [13]. Note that these shortcomings may possibly be avoided or relaxed via boundary layer approach, integration, and some filtering techniques.

In a typical sliding mode control design, a switching controller with high enough gain is applied to suppress the effects of modeling uncertainties and disturbances, and the agent dynamics are forced to move along a stabilizing manifold, which is also called a *sliding manifold*. The value of the gain is computed using the known bounds on the uncertainties and disturbances.

Next, we design a sliding mode control scheme to solve Problem 1. In [3] it was proved that if the agents are forced to move according to equation

$$\dot{p}_i = -\sigma \nabla_{p_i} J(p_1, \ldots, p_N, p_T) - \beta \text{sgn}(\nabla_{p_i} J(p_1, \ldots, p_N, p_T)) \tag{14}$$

where $\beta \geq \beta_{T_v}$, the time derivative of J becomes smaller than zero $\dot{J} \leq 0$ and potential function J converges to a minimum. However, there is one drawback of this method. The time derivative of the $\text{sgn}(\nabla_{p_i} J(p_1, \ldots, p_N, p_T))$ function is infinite at the instances when $\nabla_{p_i} J(p_1, \ldots, p_N, p_T) = 0$. In order to overcome this problem similar to [3] we pass the nonsmooth $sgn(\nabla_{p_i} J(p_1, \ldots, p_N, p_T))$ signal through a low-pass filter to extract its average or equivalent value. Therefore, instead of using directly $\text{sgn}(\nabla_{p_i} J(p_1, \ldots, p_N, p_T))$, we use the output of the second order low-pass filter in equation (14) . The reason of using a second order filter instead of the first order filter of [3] will be apparent by the end of this section. Let us define the dynamics of the filter as

$$\mu^2 \ddot{q}_i = -\mu \dot{q}_i - q_i + \beta \text{sgn}(\nabla_{p_i} J(p_1, \dots, p_N, p_T)). \quad (15)$$

Here μ is a small positive constant which must be properly chosen. Also as mentioned above β must be chosen such that $\beta \geq \beta_{T_v}$. With proper parameter selections the output of the filter on avarage satisfies

$$q_i \approx [\beta \text{sgn}(\nabla_{p_i} J(p_1, \dots, p_N, p_T))]_{eq}$$

where $[\beta \text{sgn}(\nabla_{p_i} J(p_1, \dots, p_N, p_T))]_{eq}$ is the equivalent (avarage) component of $\beta \text{sgn}(\nabla_{p_i} J(p_1, \dots, p_N, p_T))$. The result is that although $\beta \text{sgn}(\nabla_{p_i} J(p_1, \dots, p_N, p_T))$ is not differentiable its aprroximation q_i is differentiable and can be used in the design of the sliding mode controller. By using q_i instead of $\beta \text{sgn}(\nabla_{p_i} J(p_1, \dots, p_N, p_T))$, the algorithm becomes implementable. Now returning to the design of the sliding mode controller for simplicity let us define $p_S \triangleq [p_1^\top, \dots, p_N^\top]^\top$ (the concatenated positions of the swarm members) and $p \triangleq [p_S^\top, p_T^\top]^\top$. Moreover, let

$$\nabla_{p_i} J(p) = \begin{bmatrix} J_{x_i}(p) \\ J_{y_i}(p) \end{bmatrix},$$
$$\nabla_{p_i} J_T(p) = \begin{bmatrix} J_{Tx_i}(p) \\ J_{Ty_i}(p) \end{bmatrix}, \; \nabla_{p_i} J_F(p) = \begin{bmatrix} J_{Fx_i}(p) \\ J_{Fy_i}(p) \end{bmatrix}$$

denote the gradient of the potential at p_i. From (6) we have

$$\nabla_{p_i} J(p) = \nabla_{p_i} J_T(p) + \nabla_{p_i} J_F(p)$$

where J_{Tx_i}, J_{Ty_i}, J_{Fx_i} and J_{Fy_i} can be extracted from (6), (7) and (8) as

$$J_{Tx_i} = w_T \|p_i - p_T\|^2 (x_i - x_T), \quad (16)$$
$$J_{Ty_i} = w_T \|p_i - p_T\|^2 (y_i - y_T), \quad (17)$$
$$J_{Fx_i} = w_F \sum_{j=1, j\neq i}^{N} (x_i - x_j) \left[a_{ij} - b_{ij} e^{-\frac{\|p_i - p_j\|^2}{c_{ij}}} \right], \quad (18)$$
$$J_{Fy_i} = w_F \sum_{j=1, j\neq i}^{N} (y_i - y_j) \left[a_{ij} - b_{ij} e^{-\frac{\|p_i - p_j\|^2}{c_{ij}}} \right]. \quad (19)$$

In order to achieve satisfaction of motion along the negative gradient of the potential function (i.e., equation (14)) we need

$$-\sigma \nabla_{p_i} J(p) - q_i = \begin{bmatrix} -\sigma J_{x_i}(p) - q_{x_i} \\ -\sigma J_{y_i}(p) - q_{y_i} \end{bmatrix} = \begin{bmatrix} v_i \cos(\theta_i) \\ v_i \sin(\theta_i) \end{bmatrix}. \quad (20)$$

Let

$$-Z_i \triangleq -\sigma \nabla_{p_i} J(p) - q_i \triangleq \begin{bmatrix} -Z_{x_i} \\ -Z_{y_i} \end{bmatrix}$$

In other words, we need

$$v_i = \|Z_i\|, \; \theta_i = \angle([-Z_{x_i}, -Z_{y_i}]^\top)(\text{mod } 360°) \quad (21)$$

where $\angle[x, y]^\top$ for an arbitrary vector $\angle[x, y]^\top \in \mathbb{R}$ denotes the counter-clock-wise angle form the cartesian coordinate x-axis to the vector $[x, y]^\top$.

Note that since the inputs in the agent model (3) are $u_{i1} = F_i$ and $u_{i2} = \tau_i$, i.e. v_i and θ_i cannot be applied directly, the terms

$$v_{id} \triangleq \|Z_i\|, \; \theta_{id} \triangleq \angle([-Z_{x_i}, -Z_{y_i}]^\top)(\text{mod } 360°) \quad (22)$$

need to be considered as desired set-point values for v_i and θ_i, respectively.

Our objective is to force the motion of the agents such that the differences $|v_i - v_{id}|$ and $|\theta_i - \theta_{id}|$ converge to zero. With this objective in mind, similar to [20], [21], [27], let us define two sliding surfaces as in [24], one for the translational speed v_i and one for the orientation θ_i, respectively, as

$$s_{v_i} = v_i - v_{id} \quad (23)$$
$$s_{\theta_i} = c_\theta (\dot{\theta}_i - \dot{\theta}_{id}) + (\theta_i - \theta_{id}), \quad (24)$$

where $c_\theta > 0$ is a positive constant, and v_i and θ_i are the actual linear and angular speeds, respectively, whereas v_{id} and θ_{id} are the desired linear and angular speeds as defined in (22). With these definitions, our objective becomes to design the control inputs u_{i1} and u_{i2} so that $s_{v_i} \to 0$ and $s_{\theta_i} \to 0$ asymptotically, since if they are achieved we will have $v_i \to v_{id}$ and $\theta_i \to \theta_{id}$.

The existence of the additional term $c_\theta(\dot{\theta}_i - \dot{\theta}_{id})$ in (24) comes from the double integrator relationship between the terms θ_i and $u_{i2} = \tau_i$.

It is well known from the sliding mode control theory that if we have the reaching conditions

$$s_{v_i}\dot{s}_{v_i} \le -\varepsilon_1|s_{v_i}| \tag{25}$$

$$s_{\theta_i}\dot{s}_{\theta_i} \le -\varepsilon_2|s_{\theta_i}| \tag{26}$$

satisfied for some constants $\varepsilon_1, \varepsilon_2 > 0$, then $s_{v_i} = 0$ and $s_{\theta_i} = 0$ will be achieved in finite time [12], [13].

In order to achieve the satisfaction of (25) we choose the first control input $u_{i1} = F_i$ as

$$u_{i1} = -K_{i1}\mathrm{sgn}(s_{v_i}) \tag{27}$$

with which the time derivative of s_{v_i} becomes

$$\dot{s}_{v_i} = -\frac{K_{i1}}{m_i}\mathrm{sgn}(s_{v_i}) + \frac{1}{m_i}f_{v_i} - \dot{v}_{id}$$

and we have

$$\begin{aligned} s_{v_i}\dot{s}_{v_i} &= s_{v_i}\left(-\frac{K_{i1}}{m_i}\mathrm{sgn}(s_{v_i}) + \frac{1}{m_i}f_{v_i} - \dot{v}_{id}\right) \\ &= -\frac{K_{i1}}{m_i}|s_{v_i}| + \frac{1}{m_i}s_{v_i}f_{v_i} - s_{v_i}\dot{v}_{id} \\ &\le -\left(\frac{K_{i1}}{\overline{M}} - \frac{1}{\underline{M}}f_v^+ - \sigma\alpha_1(p) - \frac{2\beta}{\mu}\right)|s_{v_i}| \end{aligned} \tag{28}$$

where $(\alpha_1(p) \ge 0)$ and $\left(\frac{2\beta}{\mu}\right)$ form the upper bound for $|\dot{v}_{id}|$; ($|\dot{v}_{id}| \le \alpha_1(p) + \frac{2\beta}{\mu}$). The derivation of $\alpha_1(p)$ is shown explicitly in the sequel. Then by choosing K_{i1} according to

$$K_{i1} \ge \frac{\overline{M}}{\underline{M}}\left(\sigma\underline{M}\alpha_1(p) + \underline{M}\varepsilon_1 + \underline{M}\frac{2\beta}{\mu} + f_v^+\right) \tag{29}$$

one guarantees that (25) is satisfied and sliding mode occurs (i.e., $s_{v_i} = 0$ is satisfied) in finite time.

The existence and properties of $\alpha_1(p)$ depend on the properties of the potential function, which is chosen by the designer. In other words, one can choose the potential function such that such $\alpha_1(p)$ exists. In [24], the value of $\alpha_1(p)$ was found for a similar potential function. That derivation can be expanded for the potential function (6) of this work in order to find a value for $\alpha_1(p)$. The outcome of the derivation is given by

$$\alpha_1(p) = 2\bar{\alpha}(p)\left[\max_{i\in\{1,\dots,N\}}\left(\sum_{j=1,j\neq i}^{N}\|G_F(p_i - p_j)\|\right) + \max_{i\in\{1,\dots,N\}}\left(\|G_T(p_i - p_T)\|\right)\right] \tag{30}$$

where

$$\bar{\alpha}(p) = \max_{k\in\{1,\dots,N\}}\left(\|\sigma\nabla_{p_k}J(p)\| + \beta + |s_{v_k}(0)|\right),$$

$$G_F(p_i - p_j) = a_{ij}I + b_{ij}\exp\left(-\frac{\|p_i - p_j\|^2}{c_{ij}}\right)\left(\frac{2}{c_{ij}}(p_i - p_j)(p_i - p_j)^\top - I\right),$$

and

$$G_T(p_i - p_T) = 2(p_i - p_T)(p_i - p_T)^\top + \|p_i - p_T\|^2 I.$$

Similarly, for the second sliding surface choosing the control input as

$$u_{i2} = -K_{i2}\mathrm{sgn}(s_{\theta_i}) \tag{31}$$

the time derivative of s_{θ_i} becomes

$$\dot{s}_{\theta_i} = -c_\theta\frac{K_{i2}}{I_i}\mathrm{sgn}(s_{\theta_i}) + \frac{c_\theta}{I_i}f_{w_i} - c_\theta\ddot{\theta}_{id} + \omega_i - \dot{\theta}_{id} \tag{32}$$

and we have

$$\begin{aligned} s_{\theta_i}\dot{s}_{\theta_i} &= s_{\theta_i}\left(-\frac{c_\theta K_{i2}}{I_i}\operatorname{sgn}(s_{\theta_i}) + \frac{c_\theta}{I_i}f_{w_i} - c_\theta\ddot{\theta}_{id} + \omega_i - \dot{\theta}_{id}\right) \\ &\leq -\left(\frac{c_\theta K_{i2}}{\overline{I}} - \frac{c_\theta}{\overline{I}}f_w^+ - c_\theta|\ddot{\theta}_{id}| - |\dot{\theta}_{id}| - |\omega_i|\right)|s_{\theta_i}| \end{aligned}$$

By choosing K_{i2} as

$$K_{i2} \geq \frac{\overline{I}}{c_\theta}\left(\frac{c_\theta}{\underline{I}}f_w^+ + c_\theta\Gamma_2(p) + |\dot{\theta}_{id}| + |\omega_i| + \varepsilon_2\right), \tag{33}$$

where $\Gamma_2(p)$ is a computable bound (discussed below) such that $|\ddot{\theta}_{id}| \leq \Gamma_2(p)$, one can guarantee that (26) is satisfied and the second sliding surface $s_{\theta_i} = 0$ in (24) will as well be reached in finite time.

In order to be able to compute the value of s_{θ_i} one needs the time derivative of θ_{id}, which is given by

$$\begin{aligned} \dot{\theta}_{id} &= \frac{\frac{d}{dt}\left(\frac{Z_{y_i}}{Z_{x_i}}\right)}{1 + \left(\frac{Z_{y_i}}{Z_{x_i}}\right)^2} \\ &= \frac{\frac{d}{dt}\left(Z_{y_i}\right)\cdot Z_{x_i} - \frac{d}{dt}\left(Z_{x_i}\right)\cdot Z_{y_i}}{\left(Z_{x_i}\right)^2\left(1 + \left(\frac{Z_{y_i}}{Z_{x_i}}\right)^2\right)} \\ &= \frac{\frac{d}{dt}\left(Z_{y_i}\right)\cdot Z_{x_i} - \frac{d}{dt}\left(Z_{x_i}\right)\cdot Z_{y_i}}{\left(Z_{x_i}\right)^2 + \left(Z_{y_i}\right)^2}. \end{aligned} \tag{34}$$

Now, in order to write (34) explicitly one needs not only the equations (16), (17), (18) and (19) but also the time derivatives of them. These derivatives can be computed as

$$\frac{d}{dt}\left(J_{Tx_i}\right) = w_T\left[\left[3(x_i - x_T)^2 + (y_i - y_T)^2\right](\dot{x}_i - \dot{x}_T) + 2(x_i - x_T)(y_i - y_T)(\dot{y}_i - \dot{y}_T)\right],$$

$$\frac{d}{dt}\left(J_{Ty_i}\right) = w_T\left[\left[(x_i - x_T)^2 + 3(y_i - y_T)^2\right](\dot{y}_i - \dot{y}_T) + 2(x_i - x_T)(y_i - y_T)(\dot{x}_i - \dot{x}_T)\right],$$

$$\begin{aligned} \frac{d}{dt}\left(J_{Fx_i}\right) = w_F\textstyle\sum_{j=1,j\neq i}^{N}\Bigg[&-\left[a_{ij} - b_{ij}\left(1 - \frac{2(x_i-x_j)^2}{c_{ij}}\right)\exp\left(-\frac{\|p_i-p_j\|^2}{c_{ij}}\right)\right](\dot{x}_i - \dot{x}_j) \\ &+\left[b_{ij}\frac{2(x_i-x_j)(y_i-y_j)}{c_{ij}}\exp\left(-\frac{\|p_i-p_j\|^2}{c_{ij}}\right)\right](\dot{y}_i - \dot{y}_j)\Bigg] \end{aligned}$$

$$\begin{aligned} \frac{d}{dt}\left(J_{Fy_i}\right) = w_F\textstyle\sum_{j=1,j\neq i}^{N}\Bigg[&-\left[a_{ij} - b_{ij}\left(1 - \frac{2(y_i-y_j)^2}{c_{ij}}\right)\exp\left(-\frac{\|p_i-p_j\|^2}{c_{ij}}\right)\right](\dot{y}_i - \dot{y}_j) \\ &+\left[b_{ij}\frac{2(x_i-x_j)(y_i-y_j)}{c_{ij}}\exp\left(-\frac{\|p_i-p_j\|^2}{c_{ij}}\right)\right](\dot{x}_i - \dot{x}_j)\Bigg]. \end{aligned}$$

As mentioned above, the bound ($\Gamma_2(p) \geq |\ddot{\theta}_{id}|$) on $|\ddot{\theta}_{id}|$ is needed in order to determine the controller gain K_{i2}. Similar to the case with $|\dot{v}_{id}|$, one can compute this bound as

$$\Gamma_2(p) = \frac{\sigma\alpha_2(p) + 2\beta(\frac{\mu+1}{\mu^2})}{\|Z_i\|} + 2(\Gamma_1(p))^2,$$

where $\left(2\beta(\frac{\mu+1}{\mu^2})\right)$ is the bound on $\|\ddot{q}\|$ and $\Gamma_1(p)$ is given by

$$\Gamma_1(p) = \frac{\sigma\alpha_1(p) + 2(\frac{\beta}{\mu})}{\|Z_i\|},$$

(here $\alpha_1(p)$ is the bound given by (30)) and $\alpha_2(p)$ is the bound on $\left\|\frac{d^2}{dt^2}(\nabla_{p_i}J(p))\right\|$ which can be calculated as

$$\left\| \frac{d^2}{dt^2}(\nabla_{p_i} J(p)) \right\| \leq \alpha_2(p) \;=\; 2\bar{\alpha}(p)\left[\max_{i\in\{1,\ldots,N\}} \|\dot{G}_T(p_i - p_T)\| + \max_{i\in\{1,\ldots,N\}} \sum_{j=1, j\neq i}^{N} \|\dot{G}_F(p_i - p_j)\| \right]$$
$$+\; 2\max\left(\max_{i\in\{1,\ldots,N\}} \left(\frac{K_{i1}}{\underline{M}} + \frac{f_v^+}{\underline{M}} + |v_i||w_i| \right), \beta_{T_a} \right) \max_{i\in\{1,\ldots,N\}} \|G_T(p_i - p_T)\|$$
$$+\; 2 \max_{i\in\{1,\ldots,N\}} \left(\frac{K_{i1}}{\underline{M}} + \frac{f_v^+}{\underline{M}} + |v_i||w_i| \right) \max_{i\in\{1,\ldots,N\}} \sum_{j=1, j\neq i}^{N} \|G_F(p_i - p_j)\|.$$

The derivatives $\dot{G}_T(p_i - p_T)$ and $\dot{G}_F(p_i - p_j)$ in the above equation can be computed as

$$\dot{G}_T(p_i - p_T) \;=\; 4(\dot{p}_i - \dot{p}_T)(p_i - p_T)^\top + 2(p_i - p_T)^\top(\dot{p}_i - \dot{p}_T)I,$$

and

$$\dot{G}_F(p_i - p_j) = -2\frac{b_{ij}}{c_{ij}} \exp\left(-\frac{\|p_i - p_j\|^2}{c_{ij}} \right) \left[2(\dot{p}_i - \dot{p}_j)^\top (p_i - p_j) \left(\frac{2}{c_{ij}}(p_i - p_j)(p_i - p_j)^\top - I \right) - 2(\dot{p}_i - \dot{p}_j)(p_i - p_j)^\top \right].$$

One drawback of the algorithm is that for its implementation, each agent A_i needs not only the position but also the velocity of its neighbors (which are all the other agents in the particular setting here - but this is not necessarily required to be the case in general). Note here that one way of obtaining information about the velocity $\dot{p}_j$ of another agent A_j, in case $\dot{p}_j$ is not measurable is to estimate this velocity via interpolation of the current and past measurements of the position p_j.

We would like to emphasize that the procedure based on the sliding mode control technique discussed above will guarantee proper behavior in the presence of uncertainties in the mass m_i and the inertia I_i of the robots and additive disturbances f_{v_i} and f_{w_i} to the linear and angular speed dynamics which constitute very realistic assumptions.

Once the sliding mode occurs on all the surfaces (which happens in finite time) and agents start to move according to (14) and potential function (6) is chosen such that it satisfies (13) and (10), then we know that Problem 1 will be solved. One issue to note, however, is that after occurrence of sliding mode we reach $v_i = v_{id}$ but not necessarily $\theta_i = \theta_{id}$. In fact, after occurrence of sliding mode we have $\theta_i \to \theta_{id}$ exponentially fast and the speed of convergence depends on the slope of the sliding surface $-\frac{1}{c_\theta}$. Therefore, one needs to choose c_θ as small as possible in order to achieve faster convergence and avoid any instabilities. Note also that decreasing the parameter c_θ will require increasing the controller gain K_{i2}.

IV. Simulation Results

In this section we present simulation results that test the effectiveness of the control scheme proposed in the previous sections. In implementation of this control scheme, we have used the potential function (6), (7), (8) with $w_T = 0.4$ and $w_F = 2.25$. The parameters of the second order filter (15) and for the equation (14) are chosen as $\sigma = 0.25$, $\beta = 2$ and, $\mu = 0.1$.

The target is assumed move in $\mathbb{R}^2$ with the dynamics

$$\begin{aligned} \dot{x}_T(t) &= 0.25(m), \\ \dot{y}_T(t) &= \sin(0.25t)(m), \end{aligned}$$

where $p_T(t) = [x_T(t), y_T(t)]^\top$. The bounds on the unknown mass and inertia of the agents are taken as $\overline{M} = \overline{I} = 4$ and $\underline{M} = \underline{I} = 1$. The bounded unmodeled dynamics and disturbances are assumed as

$$f_{v_i}(t) = f_{w_i}(t) = 1.5\sin(1.5t)$$

and the corresponding known bounds on them are chosen as $f_v^+ = f_w^+ = 1.5$.

The desired formation, as shown in Figure 2, is a complete K_4 graph framework with the desired inter-agent distances

$$d_{12} = d_{14} = d_{34} = d_{32} = d_{24} = 2,\; d_{13} = 2\sqrt{3}\ (\text{m}).$$

In order for the potential function to have a minimum at these desired distances, its parameters are selected as $b_{ij} = 20$ and $c_{ij} = 10$ for all i, and the corresponding a_{ij} is calculated according to equation (10) as

$$a_{ij} = b_{ij} \exp\left(-\frac{(d_{ij})^2}{c_{ij}} \right).$$

The slope parameter for the s_{θ_i} surface is chosen as $c_\theta = 0.05$ and the sliding mode gains are calculated at every step according to inequalities (29) and (33). The sgn function which is used in the calculation of the control inputs seems to work well in theory. However, in practice it may create numerical problems and also cause high frequency chattering because of its discontinuous characteristic. Instead of the sgn function, we used the function $\tanh(\gamma y)$, where γ is a smoothness parameter

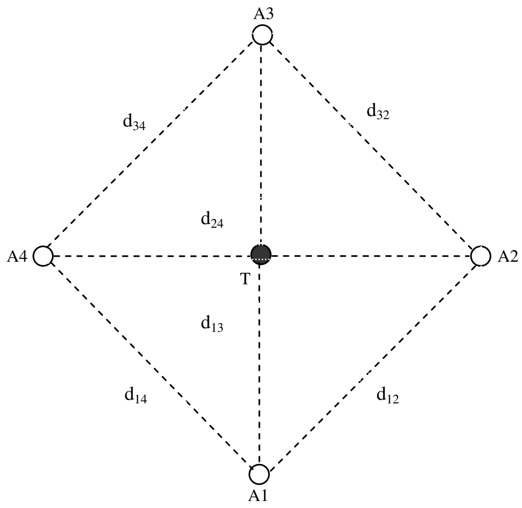

Fig. 2. Desired formation.

which determines the slope of the function around $y = 0$ and therefore the similarity between the sgn and $\tanh$ functions. The smoothness parameter in our case is chosen as $\gamma = 40$.

Figure 3 shows the paths of the swarm members and the target. It is observed that with random initial positions the swarm members quickly form the desired geometrical shape and track the target such that the target is surrounded/enclosed by the agents in the swarm. This implies that the target is within the convex hull formed by the positions of the agents and that $p_T(t) \rightarrow \mathrm{conv}\{p_1(t), \ldots, p_N(t)\}$ in finite time. It is observed that when $w_T \cong w_F$ the tracking agents try to keep the target at the center of the diamond but this time it becomes more difficult for the agents to keep the desired inter-agent distances.

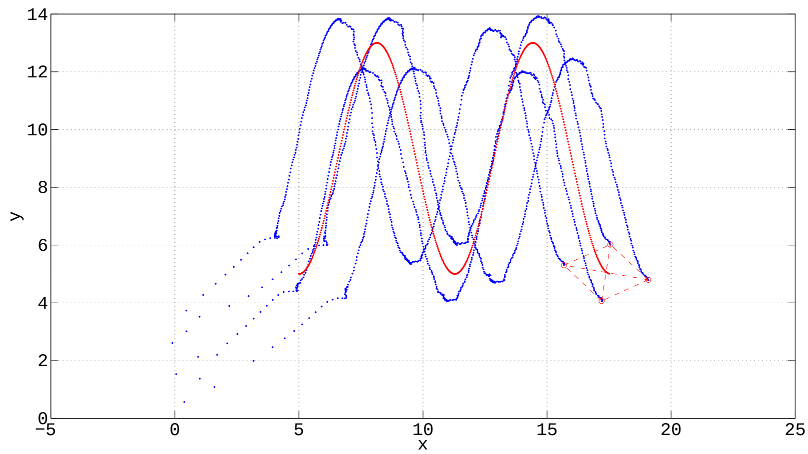

Fig. 3. Paths of Swarm Members.

Figure 4 illustrates the satisfaction of equation (5) in Problem 1. The simulation last 50 time units and every 0.1 time unit a data point is taken so the figure is formed of 500 data points. In the figure $e_i(t)$ represents the error in the desired inter-agent distances, i.e., $abs(\|p_i(t) - p_j(t)\| - d_{ij})$. It is observed that the differences converge to small values (close to zero) as expected.

The plots in Figures 5 and 6 show the first and the second control inputs for one of the agents. A single control signal is composed of nearly 380 thousand points at the end of the simulation but nearly after the first 2000 samples signals reach steady state and their magnitude becomes approximately zero. So the figures are plotted for the first 5000 samples. High magnitude control signals phenomenon of sliding mode can be observed from the figures. It is seen roughly from the figures that the control signal gains are bounded as $K_{i1} \leq 4000$ and $K_{i2} \leq 20000$. The aim of this paper is to prove that motion control of non-holonomic robots can be performed using sliding mode technique so decreasing the magnitude of control signals is not in current scope but it may be a subject of future work.

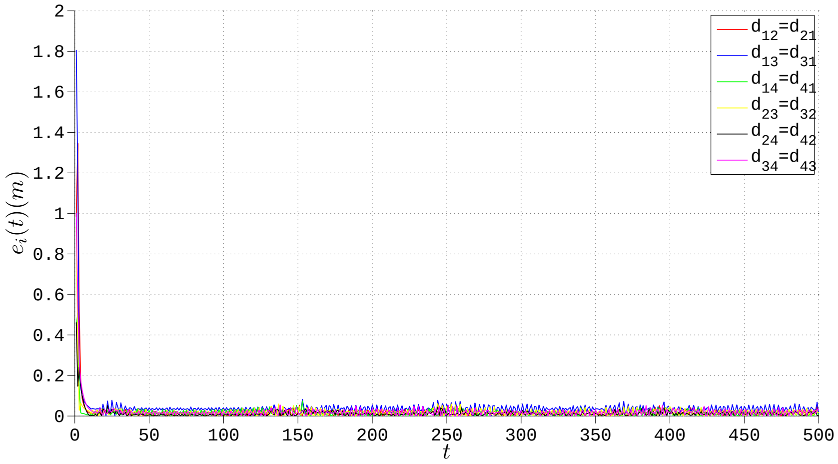

Fig. 4. Difference between inter-member distances and desired distances.

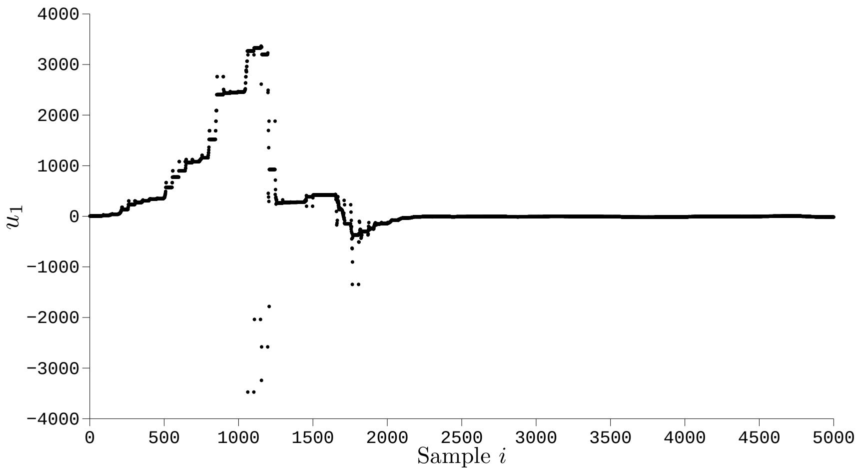

Fig. 5. First control input of one of the agents.

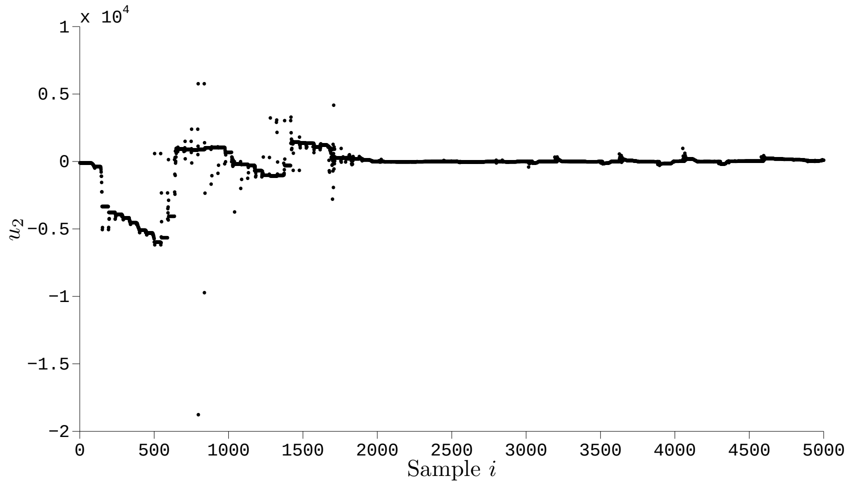

Fig. 6. Second control input of one of the agents.

V. Concluding Remarks

In this paper, tracking, capturing and enclosing of a maneuvering target with a swarm of non-holonomic agents in a pre-defined geometrical formation has been discussed. In order to meet this control goal, a decentralized control scheme based on artificial potential functions and the sliding mode control technique has been designed specifically considering agents with non-holonomic unicycle dynamic model, modeling uncertainties and some additive disturbances. It has been shown, both theoretically and via simulations, that using the proposed control scheme the swarm would capture the target and enclose it while moving in a formation with a certain predefined geometrical shape. Future research can focus on the case in which the agents can sense or communicate with only a subset of the agents in the swarm. Another topic of future research could be the examination of the system performance in the existence of sensing errors.

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