

Conceptual Design Document on hierarchical hybrid schema for global map building and localisation

Deliverable 6.2.1

First draft

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1 Overview

The aim of this document is to present the conceptual design if a hierarchical hybrid schema for global map building/up-dating and localisation in the Guardians.

2 Introduction: GUARDIANS Scenario and Map Building

The GUARDIANS major use-case scenario, proposed by the Fire and Rescue Services, consists of searching an industrial warehouse or basement (in the event of fire). A detailed description of the scenario is given in the deliverable D6.1.1/2. In this document we outline those aspects of scenario which is related to the environmental map building/updating.

The aim of environmental map up-dating is to register the positions of the robots, human beings and other points of interest on the map in order to support human operators with comprehensible and accurate information on the situation of the inspected area.

The tasks of a swarm of robots in the scenario can be roughly split out into two categories. The first category of tasks are related to the direct assistance of fire-fighters, namely of guiding a fire fighter, accompanying him/her and indicating him possible obstacles and locations of danger. The second category is the tasks, which a swarm fulfils without a human squad-leader, such as deployment on the site, positioning as beacons and maintaining communication. Some tasks of both categories are overlapped, such as searching and navigating the environment; the main difference in this respect is that in the first category the robots act within an immediate vicinity of the human, and therefore their sensor range covers only a relatively small area of the environment, whereas in the second category of tasks the robots can disperse in the site and therefore the perception of the environment will be more global.

The second category of tasks may use a wireless ad-hoc network formed by robots not only for communication purposes and broadcasting the collected data, but also for providing an initial sketch of the environment. The chain of robots equipped with wireless communication module can be viewed as a topological graph with robots as nodes and communication links as edges. The homology of this graph will give the first layout of the environment. Further exploitation of this graph by using the localisation methods proposed in WP3 (WP3.4) will transform the topological graph into a geometric graph; thus providing an initial metric sketch of the site. The positioning of some robots as beacons at key points to indicate obstacles and passages, or for facilitation of communication, is also envisaged. Such beacons will be served as landmarks for topological map building as well as for navigation purposes. In order to achieve an optimal distribution of robots on the site to 1) cover as large as possible

area, 2) reflect the specific features of interest in the environments = 'smart' positioning of beacons and 3) provide a robust communication network a dedicated algorithm, called the 'dynamic triangulation method' is under development in WP3 (WP3.8).

Therefore, there is a strong relation between map building algorithms, swarming algorithms and networking protocols. The GURDIANS robot swarm represents a self-organising system, which can be seen as hybrid of a (heterogeneous) swarm, a mobile ad-hoc network and an (evolving) topological map of the environment. Such a self-organising system, which is also applied to a very important practical problem, is to our knowledge has not been introduced before and represents one of the major innovations of the GUARDIANS projects.

The next steps in the environmental Map Building/Up-dating in GUARDIANS will be the exploitation of the sensor range of each individual robot, thus creating individual maps, and their fusion into local metric maps by combining the individual maps of robots that are within a communication range of each other.

Fusion of local maps will give us a global metric map of the environment. A detailed description of conceptual design of the hierarchical map building in GUARDIANS is given in Section 3. Section 2 briefly describes the state-of the-art in Map building, in particular in topological map building and emphasises new perspectives, challenges and innovations within this domain brought forward by the GUARDIANS project.

3 Robotic Map Building: An overview

3.1 Challenges in Map Building

The Map building problem in mobile robotics consists of creating spatial representations of the robots' environment. By environment one understands the interpretation of the robot's world, based on sensory observations (data), by means of multi-sensor reaction to `real-world' stimuli. These `real-world' observations can be of different kind: from geometric (e.g. range, size and shape) to physical (e.g. temperature, force and chemical composition).

In order to obtain a more detailed, robust and accurate picture of the real-world fusion of sensory information is performed. Fusion of sensory data can be separated into several kinds:

- fusion of data obtained from the sensors from a robot at a given time and space point, which can be interpreted as an individual map of the environment of a robot at a given time;
- fusion of individual maps of a robot at a sequence of consecutive time points;
- fusion of individual maps of several robots at a given time if several robots are deployed;
- fusion of individual maps of several robots at a sequence of consecutive time points;
- fusion of data obtained by a specific sensor at a sequence of time points.

Realisation of any kind of data fusion leads to the following inherent problems in Map Building [Sull2003]:

- How to translate sensor readings into knowledge about the environment;
- How to determine the (exact) position and orientation of the robot when it receives the sensor readings.

In order to solve these problems the following challenges in Map Building should be addressed [Thrun2002]:

- 1. Uncertainty and the nature of Measurement Noise
- 2. Dimensionality of the environment
- 3. Correspondence problem
- 4. Dynamic Environment

5. Robotic Exploration

These challenges are interrelated. The *measurement noise* problem means that whatever a robot infers about its environment is afflicted with systematic, correlated errors. Accommodating such systematic errors leads to complex, both from a mathematical and implementation point of view algorithm. This problem is directly related to the next key difficulty of the robot mapping problem, which is *high dimensionality* of the entities that are being mapped.

The *correspondence problem* is the problem of determining if sensor measurements taken at different points in time and in space correspond to the same physical object in the world.

Combining together the second and third challenge, we can say the map building becomes *high-dimensional multivariate correspondence (data association) problem.* The above mentioned problem is already very difficult under a static world assumption, when environment does not change over time.

In a real world scenario, the *environment* is, in general, *dynamic*. It can change over time: doors open or close, passages block, people move. The map that a robot built initially may, after a period of time, no longer be valid. Consequently, there is a need for the robot to be able to learn within dynamic environments.

A fifth and final challenge arises from the fact that robots must choose their way while mapping the environment. The task of generating robot motion in the pursuit of building a map is commonly referred to as *robotic exploration*. If the map is unknown or the environment is dynamic then robot motion is problematic.

To reduce computational complexity, simplifying assumptions have to be made and in some cases heuristic approaches are taken to tackle a specific issue, in particular robotic exploration. As a result, robotic mapping is often referred to as Simultaneous Localisation and Mapping (SLAM) [Stachniss2004, Tardosetal2002, Bosseetal2004]. However, we see the SLAM as only a part of a more general problem of Environmental Map Building.

3.2 Approaches in Map Building

Current approaches to localization and map building in robotics can be split up into <u>metric</u> and <u>topological paradigms</u> [Thr2002, AS02].

Approaches within the metric paradigm generate fine-grained, metric descriptions of a robot's environment.

Metric maps can be roughly separated into two categories: <u>sensorial maps</u> and <u>geometric</u> <u>maps</u>. A sensorial map is a map based on direct sensor readings and is associated with Grid Occupancy Maps, introduced in [Moravec1988, Elfec89]. The environment is divided into in general evenly spaced grids (most time two-dimensional), where each grid cell may indicate the presence of an obstacle in the corresponding region of the environment. In general, a value is attached to a cell, which reflects the degree of occupancy

There are also attempts to map representation by using irregular-spaced grids [Barfoot2003].

<u>Geometric maps</u> are characterized by two main properties: a) they are composed of the union of simple geometric primitives; b) they are equipped with basic operators to manipulate these primitives.

A <u>topological map</u> (also known as a relational map) gives an abstract and compact description of the environment, mostly in the form of discrete graph in which nodes correspond to significant, easy to distinguish places or landmarks and edges or arcs represent either path segments connecting the places or actions or action-sequences that connect neighbouring places [Thrunetal98, Modetal2004].

'Pure' metric approaches in general suffer from lack of consistent spatial representation, whereas 'pure' topological approaches are less precise and might be computationally very expensive [Fox98]. Recently schemes that provide an integration of both approaches have been introduced [TNS2003]. However, the distinction between metric and topological approaches to robotic map building is rather vague, since virtually all topological approaches use geometric information [Thrun2005].

Many metric methods have the goal of building a global metrical map of the environment within a single frame of reference [Eli2003, Gut99, Hahn2003, Mont2003]. These methods, despite of their significant improvement in the past few years, are still afflicted with the problem of cumulative distortions in the map and robot position, along with the related problem of data association and the closing of large loops [Modetal2004].

Topological methods have difficulty representing "open spaces," i.e., large expanses with few features.

Another key difficult problem in topological mapping (as well as in metric mapping) is "closing the loop", i.e. recognizing when the robot has returned to a place it has already been [Beveers2005a].

In order to avoid disadvantages of each approach as well as to better represent the environment hybrid approaches have been introduced in the last decade [Rohan00, Kuipersetal2001].

Hybrid methods consist in general of combining topological maps of the global environment with metrical maps of local spaces have been proposed. Local metrical maps in separate local frames of reference do not require global metric consistency, and their integration with a global topological map helps to overcome such weaknesses of pure topological methods as 'imprecision' [Modetal2004, Rem2004].

Spatial model of the environment can be seen as a hierarchical layering of successive representations of map data, where each successive representation is more abstract than the previous one (and is also constructed from the observations from the less abstract representation).

3.3 New perspectives in Map Building in GUARDIANS

Also most of the previous work in robotic map building dealt only with a single robot. Recently several works have appeared that investigate multiple robots Map building. Most of them tackle this problem in theoretical and controlled environments, and even then the number of robots is in general not larger than three, with the exception of the Project Centibots developed in USA, in which it was reported that five robots were used to create a map of an environment (http://www.ai.sri.com/centibots/papers/MR-workshop-2003.pdf.)

In GUARDIANS project a swarm of robots is applied, which will be used to create the map of the environment dynamically and avoid known drawbacks in Map building. A knowledge database will be built from the pieces of information gathered by each of the robots, thus the environment is explored in a distributed manner. The use of a group of robots will produce new algorithms not feasible for a single robot, which can reduce uncertainty in map building. For example, it can drastically reduce cumulative sensorial errors, hence; improve accuracy of sensor readings, for example, odometry.

It will also contribute to a solution to the two famous problems of topological map building: the problem of 'open spaces', as well as 'closing loops'.

A new hierarchical hybrid schema that merges together the morphology of a wireless sensor network formed by a swarm, its global geometry based on the dynamic triangulation method to be developed in WP3 and local geometry based on the information gathered by the sensors of each individual robot, is envisaged to be created by the GUARDIANS project.

The schema consists of several layers; each layer reflects a certain aspect of the surroundings and differs in detail.

4 Approach to the MAP Building/Up-Dating in Guardians

A new hierarchical structure for the Map Building/Up-dating in the GUARDIANS project has been proposed.

The structure consists of several inter-related layers, and one of the envisaged innovations is that the initial layer represents a purely topological sketch of the environment, which is obtained by the exploration of the morphology of the sensor network formed by the swarm.

Shortly, the hierarchical structure of the Map Creation Framework will consist of the following layers:

- 1. <u>Initial topological layer</u>: This layer is determined by nodes and their connections in the ad-hoc wireless sensor network formed by the swarm;
- 2. <u>Initial (global) metric layer:</u> This layer is determined by localizing the positions of robots within the dynamic triangular network built by the swarm. The topological graph of the previous layer will become a geometric graph, possibly with some uncertainty in some of its regions.
- 3. <u>Local (metric) maps layer:</u> Local 2D metric maps (occupancy grids, possibly irregular), obtained on the base of the sensorial information gathered by its node in the sensor networks (individually by each robot).
- 4. <u>Skeletons layer</u>. Here possibilities to obtain skeletons directly from the network built by robot, or to use local (global) metric maps for build it on the top of them, will be explored.
- 5. <u>(Global) Topological map</u>: is built by integrating structures, obtained in the previous layers, namely the layers 1 and 4.
- 6. <u>Global 2D metric map:</u> is obtained by the fusion of local maps.
- 7. <u>3D local maps</u> of the environment (combination of sensorial maps and geometric maps).
- 8. <u>Global sketch of the environment :</u> This layer will be a fusion of the global topological map and local metric (mostly 2D) maps at the point of interest (such as obstacles, opening etc)

- 9. <u>3D global representation of the environment:</u> Fusion of 3D local maps
- 10. <u>Semantic layer:</u> this layer enhance will enhance the layer 8 with semantic information.

the layers 6, 7, 9 and 10 may be optional, in particular the layer 7, as the first five layers and their fusion (layer 8) may be sufficient to fulfil the goals related to map building in the GUARDIANS. 3D representation of the environment might be also required sparsely, i.e. only in certain locations where a three-dimensional scene reconstruction can be considered useful or necessary. For example, it can be done at certain key points of the environment to provide a unique landmark, or when a more detailed investigation of the environment is required in the case of suspected causality.

Below some of the proposed layers are depicted in more detail.

4.1 Initial topological layer and initial global metric layer

The sensor wireless (communication) network formed by a swarm provides a unique opportunity to create a topological representation of the environment without using metric information. Moreover, it can also provide a reference metric structure for building metric maps by using innovative localisation methods and swarming algorithms for an optimal distribution of the swarm.

In the last few years many works appeared dedicated to the study networks with a large number of sensor nodes. The sensor nodes are scattered in a geometric environment with nearby nodes communicating with each other directly.

Among problems addressed we can distinguish the problems of *self-localisation* and *morphology of the sensor field*.

The self-localisation problem means that the nodes of the network by using, for example, some ranging devices, estimate distances (and angles) between the neighbouring nodes and then on the basis of these measurements derive their global positions (this will provide an initial metric layer, layer 2).

The sensor networks represent *self-embedded* systems, and the problem of self-localisation is essentially the problem of *graph embedding*.

The latter problem has been well-known in the theory of graphs, computational geometry, and distance geometry; but within the domain of wireless sensor networks it has not yet got an adequate solution due to the fact that current localisation methods often fail.

The problem of the morphology of the sensor field deals with the field layout, it identifies such features as boundaries, holes, narrow passages/bridges by using only local connectivity information (homology of a graph).

The deployment of wireless sensor network is constricted by geometry of the environment. Consequently, the topology and geometry of the network can provide useful information about the geometric layout of the environment.

Roughly speaking, localisation problem deals with determination of the global geometry of the network, whereas morphological analysis specifies its topology.

Understanding the global topology of a wireless sensor network is also very important in designing basic networking functionalities, such as point-to-point routing and data gathering mechanisms.

In most scenarios, the sensor nodes are assumed to be static, and often are considered to be uniformly and densely distributed in a simple geometric environment.

In the GUARDIANS the problems of localisation and network layout find a new dimension due to the following factors:

- First of all, the sensor nodes are not static but mobile.
- The geometry of the environment is not known a priori and might be very complex.
- The network formed by the nodes is sparse due to the limited amount of the robots and the fact that the site where the robots to be deployed may be very vast.
- The latter fact also means that only a part of the environment can be covered by the network at a certain time interval.

• Some of the robots can be positioned themselves as beacons; therefore the network will be combination of mobile and static nodes.

Another difference with the recent studies on sensor networks is that the GUARDIANS network is 'building up' while robots are deployed, and not given *a priori*. This represents a new challenge but also a new opportunity to design the network structure according to our needs.

One of the methods, developing in the GUARDIANS to address the aforementioned problems, is called 'the dynamic triangulation' (WP3).

Below we briefly outline the ideas behind the method and its expected outputs.

A robot team can be seen as a graph, whose nodes are the robots, whereas the edges reflect interactions between the robots. In the communication mode, the nodes are identified with the communication nodes (routers) and the edges are wireless links.

The GUARDIANS robots are relatively 'simple' robots with a number of simple interactions. They equipped with a collection of various sensors such as camera, laser range finders, infrared, and therefore the interactions have their limitations due to the sensory ranges as well as restrictions on distances by which robots are able to communicate among themselves.

The robots can be considered 'identical' in the sense as they possess the same sensors, the same communication protocol and the same set of basic behaviours. A generalisation of this scenario might be that robots are split into several groups; each group consists of identical robots, but robots of one group may differ from the robots of another one.

Without loss of generality, in what follows we deal only with one group of robots.

Each robot has its *local communication domain,* which includes the node corresponding to a robot together with adjacent edges corresponding to the communication links of individual

robot (*individual network*). Each robot possesses also the sensory information about the surrounding environment, which can be referred to as its *domain of visibility*.

The main goal of the *dynamic triangulation method* is to deploy robots on the site in such a way as to provide its largest coverage. The robots should also be deployed in a sensible manner in order to facilitate communication and exploration of the environment.

As the geometry of the environment might be very complex, some robots can be placed as beacons at the 'openings', which might be entrances, doors, beginnings of the passages; and/or at the 'junctions', which might include the corners of obstacles.

Using the language of the graph theory we can reformulate the goal as follows: *to embed the vertices of a graph in a geometric space such that the embedded structure satisfies desired properties.*

For example, in order to form the communication infra-structure, a mesh of equilateral triangles of beacons may be desirable.

Therefore, a set of simple rules should be developed in order to distribute the robots in such a manner an equilateral triangular network would be kept whenever it is possible.

Suppose the following imaginary situation: the geometric environment is an infinite plane and we posses an infinite number of robots. In this case an equilateral mesh can easily be obtained already in the non-communicative mode just by applying only repulsive forces to the robots (within their domains of visibility). This is illustrated in the Fig. 1. The arrows indicate the local communication network for an individual robot.

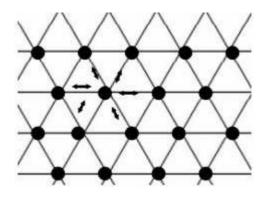


Fig. 1: Robots form an equilateral triangular lattice

Indeed, after a certain amount of time the robots system will be in balance and the robots will be situated in the vertices of an equilateral triangular lattice.

In a real situation the environment will have a boundary as well as obstacles within it, and the control algorithms applied to robots should be more sophisticated. Communication mode will provide additional tools to deploy robots in a desirable manner. However, the strength of the communication signals should be taken into account; for example, a robot should be allowed to move to its next location within the distance not larger than the half of the maximum distance by which they still can communicate. This restriction can also help with identification of obstacles situated between robots.

Individual networks form a local network that comprise several robots together with their communication links in such a way that there is a path from a robot r_i to a robot r_j along the edges of the network. So, a local network is a *connected graph*. A local network becomes a global network if all robots of the swarm are in it, otherwise the swarm may form several local networks.

Also a local network can expand when more robots are deployed, and it is dynamical, as it can move around the site.

Each local network can gather information independently, and either combine their information later, when a communication between two local networks is restored, or/and communicate the information directly to the control unit.

The topology of a local network provides us an initial information about the environment, and can be also seen an initial topological map. The network layout can indicate the boundaries of the environment as well as possible obstacles, which can lead to the initial navigation map. It can also provide a solution to the two famous problems of topological map building: the problem of 'open spaces', as well as 'closing loops'.

The aforementioned ideas are illustrated in Fig. 2.

Figure 2 shows a sketch of an environment, covered by the ad-hoc local network build by robots. Robots are represented as circles, and the communication links among them are indicated by dashed line segments. Two white circles represent the beacons positioned as the entrance to the site. Whereas other beacons can change their positions, these two positions might be preserved, as beacons at these positions can have several missions. They will provide communication between the swarm and the external facilities; serve as absolute reference points for localisation of other robots and assist robots and humans in the 'entrance beacons'; while the swarm evolves new robots can replace the acting beacons whereas the previous beacons will take part in the swarm.

The robots, which might be further positioned as beacons are indicated with rendered circles. These positions might be chosen to facilitate communication (beacons along the wall), indicate obstacles and passages, which in turn will help in map building of the environment and path planning for the human squad.

The thicker dashed lines indicate the obstacles in the environment. The part of the environment with no visible obstacles represents a triangulation. The 'grey' circles indicates the positions of the robots in the case if the environment had no obstacles, in which case an equilateral triangular grid would be preserved. Some of these positions are still possible, but not necessary, as the communication network can function without them. Some positions indicated by 'crossed' grey circles are simply impossible. The challenge here is to keep an 'optimal' network, i.e. such one that provides a good communication but not 'overcrowded' by robots, whose presence enhance neither communication nor the understanding of the environment. A larger obstacle might be detected already on the topological level by singling out areas where communication links are missing (holes in the triangulation, or cycles of k nodes in the local network, formed by the swarm, where k is larger than 3).

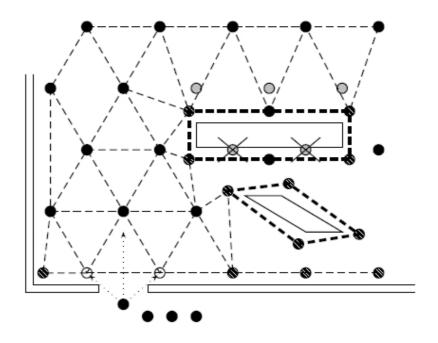


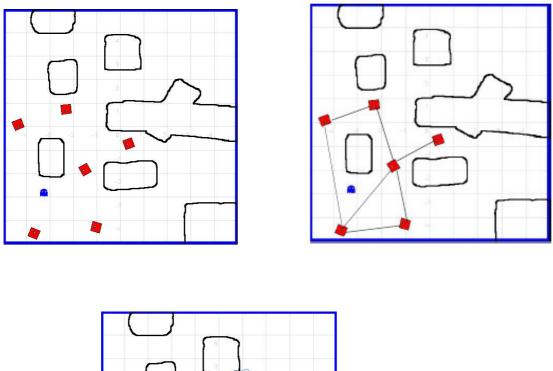
Figure 2: Dynamic triangular network schema

When new robots enter the site, they do not need to go through the whole mesh of the robots to take their positions, but take the positions of the nearest robots, which in turn, will replace some of the robots nearest to them, thus expanding the network by local replacements of nodes. In the figure three possible positions where an entering robot can move, are indicated by dotted arrows.

As robots move autonomously from the previous positions to their present positions, and there are communication links between them, the only possible locations for 'unrecognised' obstacles are within triangles formed by robots. Such obstacles can be detected by robots sensors thus enhancing the initial topological map by local metric information. Therefore, the dynamic triangulation method is the 'alloy' of swarming algorithms, networking protocols and map building.

4.2 Local metric maps layer

We illustrate the connection among the first by presenting the following three pictures (simulation by using Player/Stage).



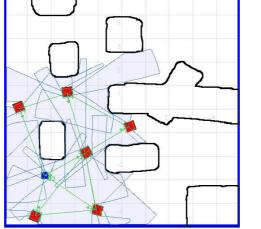


Figure 3: An example of the environment with obstacles and several robots, topological and metric layers

In the figure 3 an example of the environment with obstacles and several robots is given (upper left). In the upper right picture a topological graph formed by robots is depicted. At this level a large obstacle can be already detected. In the lower figure the topological layer is enhanced by sensorial information (layer 3), which allows to detect a small obstacle.

4.3 Global sketch of the environment enhanced by semantic layer

Here we present an example of the environmental map, at which development we are aiming.

Figure 5 depicts an environment with robots, obstacles, communication links among robots and robots/base station. Some obstacles are given in 3D form (layer 7), others in 2D. Besides this some semantic information is added.

Semantic information means that the constituents of the environment are separated into categories, such as 'places', 'objects', 'situations', and are labelled accordingly. For example, 'places' can be labelled as 'room', 'open space', 'tunnel'; 'objects' as 'door', 'person', 'robot', 'trunk', 'solid mass', etc; 'situations' as 'fire', 'dangerous gas' etc. Each semantic node is linked to corresponding metric information.

Figure 6 shows the dynamics of the environment. One of the robots moved and is 'lost'. Nevertheless, based on the dynamic triangulation methods the following information can be retrieved: the previous position of the robot and a possible current location of the robot. By moving one of the robots to the previous position of the 'lost' robot a communication link can be again established. Moreover it gives us also an indication of a new obstacle.

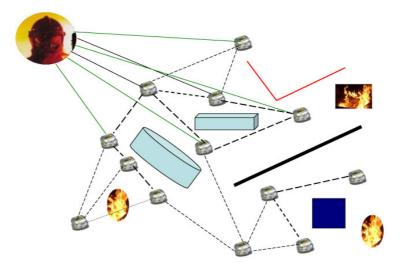


Fig. 4: Global sketch of the environment

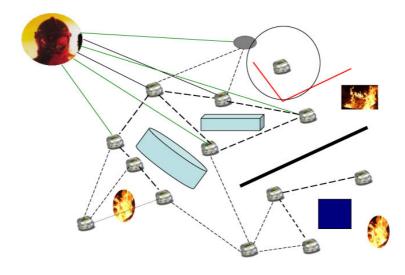


Fig. 5: Global sketch of the environment at a next point of time

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