16th Mediterranean Conference on Control and Automation
Congress Centre, Ajaccio, France
June 25-27, 2008

Tracking a Maneuvering Target with a Non-holonomic Agent Using Artificial Potentials and Sliding Mode Control

M. İlter Köksal, Veysel Gazi, Barış Fidan, and Raúl Ordóñez

Abstract— **In this article we consider tracking a maneuvering target with a non-holonomic agent. The target and the agent move in 2-dimensional space. The task is to capture/intercept the moving target using a continuous time control scheme based on artificial potentials and the sliding mode control technique. The effectiveness of the proposed control scheme is established analytically and demonstrated via a set of simulation results.**

I. Introduction

In nature, the survival of many species may critically depend on their ability to capture a prey (a target) or escape capture from a predator (a pursuer). This problem has been considered in the engineering literature where it is referred to as pursuit-evasion or sometimes target tracking problem. Recent studies on the problem of tracking a maneuvering target can be found in [1], [2], where in [1] the authors consider the problem of tracking with a single agent, whereas in [2] they consider the problem of capturing/enclosing a moving target with a swarm of agents with fully actuated (holonomic) pursuers in both cases. The model of an agent considered in [1], [2] is a general fully actuated model which can represent some omni-directional robots as well as some manipulators or even spacecraft [3]. However, in practice, most of the mobile agents (i.e., differentially driven robots, UAVs) have velocity constraints or they are underactuated and may not obey the model in [1], [2]. In this article, we consider tracking a maneuvering target with an autonomous differentially driven non-holonomic agent (sometimes called the unicycle model and described in more detail in Section II) and extend the results in [1]. Note here that the setting here constitutes more difficult setting (compared to the model in [1], [2]) in terms of controller development and the extension is not a straightforward one as the control design approach used in [1] cannot be applied directly to the non-holonomic unicycle agent dynamics model. Therefore, in this article, in order to perform a tracking task with non-holonomic agents we use the artificial potential and sliding mode control based approach in [4], where a distributed control scheme for aggregation, foraging, and formation acquisition/maintenance of swarms of non-holonomic agents was considered.

The work in this article was inspired by the earlier works of Guldner and Utkin (and their coworkers) on tracking the gradient of potential functions (potential fields) using sliding mode control [5]–[9]. Other relevant articles, mainly on potential functions based navigation, aggregation, and formation control include [1], [2], [10]–[17]. However none of these applications (other than [1]) is in the context of the particular tracking task considered here.

The paper is organized as follows. In Section II, the non-holonomic agent model is introduced and the tracking problem is defined. In Section III, the control design procedure for the solution of the tracking problem is presented in the form of a constructive analysis. In Section IV, some simulation results are presented. Finally, the paper is concluded with some final comments in Section V.

II. Agent Dynamics and Tracking Problem

Consider a system in which an agent with non-holonomic dynamics labeled as A, is required to pursue and intercept a moving target, labeled as T, under system uncertainties and additive disturbances (discussed below). Assume that agent A has the configuration depicted in Figure 1 and the equations of motion given by

$$\begin{array}{rcl} \dot{x}_A &=& v_A\cos(\theta_A), \\ \dot{y}_A &=& v_A\sin(\theta_A), \\ \dot{\theta}_A &=& w_A, \\ \dot{v}_A &=& \frac{1}{m_A}\left[F_A + f_{v_A}\right], \\ \dot{w}_A &=& \frac{1}{I_A}\left[\tau_A + f_{w_A}\right] \end{array} \tag{1}$$

where $p_A(t) = [x_A(t) \quad y_A(t)]^\top$ denotes the position of A at time instant t in Cartesian coordinates, θ_A is the steering angle, v_A is the linear speed, and w_A is the angular speed of A. The quantities m_A and I_A are positive constants and represent the mass and the moment of inertia of agent A, respectively. The control inputs for agent A are the force input F_A and the torque input τ_A. The functions f_{v_A} and f_{w_A} represent unknown additive disturbances for agent A. The disturbances are assumed to be bounded such that $|f_{v_A}| < f_v^+$ and $|f_{w_A}| < f_w^+$ for known bounds f_v^+ and f_w^+. In addition to the unknown additive system disturbances it

V. Gazi and M. İ. Köksal were supported in part by TÜBİTAK (The Scientific and Technological Research Council of Turkey) under grant No. 104E170 and by the European Commission under the GUARDIANS project (FP6 contract No. 045269). The work of B. Fidan was supported by National ICT Australia-NICTA, which is funded by the Australian Government as represented by the Department of Broadband, Communications and the Digital Economy and the Australian Research Council through the ICT Centre of Excellence program. R. Ordóñez was supported with AFRL/AFOSR grant No. F33615-01-2-3154.

V. Gazi and M. İ. Köksal are with the Dept. of Electrical and Electronics Engineering, TOBB University of Economics and Technology, Söğütözü Cad. No: 43, 06560 Ankara, TURKEY. {vgazi, i.koksal}@etu.edu.tr

B. Fidan is with National ICT Australia Ltd. and The Australian National University – Research School of Information Sciences & Engineering, Canberra, AUSTRALIA. Baris.Fidan@anu.edu.au

R. Ordóñez is with University of Dayton, Dept. Electrical and Computer Engineering, 300 College Park, Dayton, OH 45469-0232, USA, ordonez@ieee.org.

is assumed that the exact values of mass m_A and inertia I_A for agent A are unknown, however they are upper and lower bounded such that $0 < \underline{M} < m_A < \overline{M}$ and $0 < \underline{I} < I_A < \overline{I}$ and the bounds $\underline{M}, \overline{M}, \underline{I}$, and $\overline{I}$ are known.

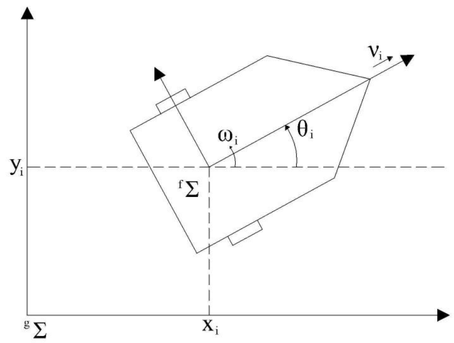

Fig. 1. Illustration of an agent with the non-holonomic unicycle dynamics.

Remark 1: In this article, all the angles including θ_A are assumed to take values within $(-\pi, \pi]$. Because of this, all addition operations on the angles are performed (mod 2π) with $-\pi$ radians shift. For example, $\theta_1 + \theta_2$ and $\theta_1 - \theta_2$ are calculated as $[(\theta_1 + \theta_2 + \pi)(\text{mod } 2\pi) - \pi]$ and $[(\theta_1 - \theta_2 + \pi)(\text{mod } 2\pi) - \pi]$, respectively. Similarly, $\dot{\theta}_A(t)$ will be defined as

$$\dot{\theta}_A(t) = \lim_{\Delta t \to 0} \frac{(\theta_A(t) - \theta_A(t - \Delta t) + \pi)(\text{mod } 2\pi) - \pi}{\Delta t}.$$

Our objective in this paper is to track a maneuvering target T, with a non-holonomic agent A, with the dynamics given in (1). In other words, we would like to design the control inputs $u_1 = F_A$ and $u_2 = \tau_A$ such that agent A follows target T and intercepts it. Letting $p_T(t) = [x_T(t) \quad y_T(t)]^\top$ denote the position of the target, at a certain time instant t, we can formulate the problem as follows.

Problem 1: (Single Tracking Problem) Consider a non-holonomic agent A which has motion dynamics given by (1), and a moving target T. Assume that, at any time instant t, A can sense its own position $p_A(t)$ and position $p_T(t)$ of T. Also, assume that velocity $\dot{p}_T(t)$ of T is known and bounded such that $\|\dot{p}_T(t)\| \leq \beta_{T_v}$, however, the acceleration of T is unknown and bounded such that $\|\ddot{p}_T(t)\| \leq \beta_{T_a}$ for known bounds β_{T_v} and β_{T_a}. With the above assumptions at hand the aim is to design the control inputs

$$u = [u_1 \quad u_2]^\top = [F_A \quad \tau_A]^\top$$

for agent A such that the inequality

$$\lim_{t \to \infty} \|p_A(t) - p_T(t)\| \leq \epsilon, \tag{2}$$

where $\epsilon > 0$ is a small design constant, is satisfied.

Note here that for solving Problem 1, we need the relative position of the target and not its actual position. In other words, we need the value of $(p_A(t) - p_T(t))$ and not the value of $p_T(t)$. Also, although knowing the velocity of the target is a strong assumption, there are methods to estimate the relative velocity $\dot{p}_A - \dot{p}_T$, such as using vision-based techniques, which can be used by the method considered here.

We approach Problem (1) using artificial potentials and sliding mode control. In the next section we discuss the development of the controller to solve problem (1).

III. Control Design

A. Artificial Potential Functions

With the objective of satisfying the requirement in (2), artificial potential functions are used in order to construct an attractive force between the tracking agent and the target. The potential function that will be used in this work needs to be a function of the positions of the agent and the target or a function of the distance between them. Also the potential function is required to have a unique minimum at $p_A = p_T$. Similar to [1] in this article we use

$$J(p_A, p_T) = J(\|p_A - p_T\|) = \frac{1}{2}\|p_A - p_T\|^2 \tag{3}$$

as the potential function between the agent and the target. This function, which can be viewed also as a Lyapunov function, is chosen among other possibilities because of its simplicity. However, other functions are also possible.

In order to satisfy (2), the chosen potential function needs to be a decreasing function of time. The time derivative of J is

$$\dot{J} = \nabla_{p_A} J^\top(p_A, p_T)(\dot{p}_A - \dot{p}_T) \tag{4}$$

which is obtained from the fact that

$$\nabla_{p_A} J(p_A, p_T) = -\nabla_{p_T} J(p_A, p_T).$$

If, similar to [1], agent A is forced to move according to equation

$$\dot{p}_A = -\sigma \nabla_{p_A} J(p_A, p_T) - \beta \text{sgn}(\nabla_{p_A} J(p_A, p_T)) \tag{5}$$

where σ and β are constants such that $\beta \geq \beta_{T_v}$ (β is greater than the bound on the velocity of the target) and $\sigma > 0$, then we have

$$\begin{aligned} \dot{J} \leq \; & - \; \sigma \|\nabla_{p_A} J(p_A, p_T)\|_2^2 \\ & - \; \beta \|\nabla_{p_A} J(p_A, p_T)\|_1 + \beta_{T_v} \|\nabla_{p_A} J(p_A, p_T)\|_1, \end{aligned}$$

which implies that

$$\dot{J} \leq -\sigma \|\nabla_{p_A} J(p_A, p_T)\|_2^2. \tag{6}$$

This equation guarantees that the artificial potential $J \geq 0$ is decreasing. As the minimum of J is at $\nabla_{p_A} J(p_A, p_T) = p_A - p_T = 0$, this implies as $t \to \infty$, $J \to 0$, $\dot{J} \to 0$ and $\nabla_{p_A} J(p_A, p_T) \to 0$. However, there is one drawback of this method. The time derivative of the $\text{sgn}(\nabla_{p_A} J(p_A, p_T))$ function is unbounded at the instances when $\nabla_{p_A} J(p_A, p_T)$ switches sign. (The time derivative of the right hand side of (5) is needed for the development of the sliding mode controller as will be discussed later in this paper.) In order to overcome this problem we use a continuous and differentiable approximation of the sgn function. In other words, we require the motion dynamics of agent A to satisfy

$$\dot{p}_A = -\sigma \nabla_{p_A} J(p_A, p_T) - \beta h(\nabla_{p_A} J(p_A, p_T)) \quad (7)$$

instead of the dynamics in (5). Here, for a scalar $k_i \in \mathbb{R}$ the function $h : \mathbb{R} \to \mathbb{R}$ is given by

$$h(k_i) = \begin{cases} -1 & , k_i < -\bar{\epsilon} \\ \sin\left(\frac{\pi k_i}{2\bar{\epsilon}}\right) & , |k_i| \le \bar{\epsilon} \\ 1 & , k_i > \bar{\epsilon} \end{cases} \quad (8)$$

where $\bar{\epsilon} > 0$ is a small constant. Similarly, for a vector $k \in \mathbb{R}^n$ such that $k = [k_1 ... k_n]^\top$ the function $h : \mathbb{R}^n \to \mathbb{R}^n$ is defined as $h(k) = [h(k_1)..h(k_i)..h(k_n)]^\top$. Note that this function introduces a boundary layer and has the same behavior as the sgn function outside the interval $[-\bar{\epsilon} \quad \bar{\epsilon}]$. This, on the other hand, implies that, when the components of the gradient $\nabla_{p_A} J(p_A, p_T)$ are outside the interval $[-\bar{\epsilon} \quad \bar{\epsilon}]$, equations (5) and (7) are equivalent implying that (6) is satisfied and the potential J is decreasing. In contrast, when the components of the gradient $\nabla_{p_A} J(p_A, p_T)$ are within the interval $[-\bar{\epsilon} \quad \bar{\epsilon}]$ the time derivative $\dot{J}$ of the potential function becomes

$$\begin{aligned} \dot{J} \le & - \sigma \|\nabla_{p_A} J(p_A, p_T)\|_2^2 \\ & - \beta \nabla_{p_A} J^\top (p_A, p_T) \sin\left(\frac{\pi \nabla_{p_A} J(p_A, p_T)}{2\bar{\epsilon}}\right) \\ & + \beta_{T_v} \|\nabla_{p_A} J(p_A, p_T)\|_1. \end{aligned}$$

The second term on the right hand side of the equation always satisfies

$$\beta \nabla_{p_A} J^\top (p_A, p_T) \sin\left(\frac{\pi \nabla_{p_A} J(p_A, p_T)}{2\bar{\epsilon}}\right) \ge 0$$

which means the potential function is still decreasing inside the interval $[-\bar{\epsilon} \quad \bar{\epsilon}]$ to a point where the value of the gradient of the potential function becomes so small that the value of the third term above exceeds the value of the sum of the first two terms and the potential function can not be guaranteed to decrease any more. However, this region is always smaller and located componentwise inside the interval $[-\bar{\epsilon} \quad \bar{\epsilon}]$ (i.e., it is located inside the region $[-\bar{\epsilon} \quad \bar{\epsilon}]^2$). With these facts one can see that provided (7) is satisfied, the tracking problem in (2) is solved for some $\epsilon < \bar{\epsilon}$. Moreover, the tracking error can be made arbitrarily small by choosing $\bar{\epsilon}$ appropriately. Therefore, the problem has become designing appropriate controller such that (7) is satisfied. We present a sliding mode technique based controller in the next section.

B. Sliding Mode Control Design

Sliding mode control [18], [19] is a widely used technique in various application areas, including gradient tracking control of mobile robots, target tracking and multi-agent system coordination and control as mentioned in Section I. This is mainly because of its suppressive and robust characteristics against the uncertainties and the disturbances in system dynamics. The shortcomings (of the raw form of the sliding mode control scheme), on the other hand, are the so-called chattering effect and possible generation of high-magnitude control signals [18], [19]. Note that these shortcomings may possibly be avoided or relaxed via boundary layer approach, integration, and some filtering techniques.

In a typical sliding mode control design, a switching controller with high enough gain is applied to suppress the effects of modeling uncertainties and disturbances, and the agent dynamics are forced to move along a stabilizing manifold, which is also called *sliding manifold*. The value of the gain is computed using the known bounds on uncertainties and disturbances.

In this section, we present a sliding mode control scheme to solve Problem 1. For simplicity, let us define $p \triangleq [p_A^\top, p_T^\top]^\top$. Depending on the value of $\|\nabla_{p_A} J(p)\|$ two different expressions will be considered. We will refer as **Case 1** to the controller expression when $\|\nabla_{p_A} J(p)\| \neq 0$ and as **Case 2** to the controller expression when $\|\nabla_{p_A} J(p)\| = 0$.

The objective is to force the agents to move according to equation (7) for which we need

$$-\sigma \nabla_{p_A} J(p) - \beta h(\nabla_{p_A} J(p)) = \begin{bmatrix} v_A \cos(\theta_A) \\ v_A \sin(\theta_A) \end{bmatrix} \quad (9)$$

to be satisfied. Let

$$-Z \triangleq -\sigma \nabla_{p_A} J(p) - \beta h(\nabla_{p_A} J(p)) \triangleq \begin{bmatrix} -Z_x \\ -Z_y \end{bmatrix}. \quad (10)$$

In other words, to achieve (9), we need

$$v_A = \|Z\|, \ \theta_A = \angle([-Z_x, -Z_y]^\top), \quad (11)$$

where $\angle\left([x \quad y]^\top\right) \in (-\pi, \pi]$ for an arbitrary vector $[x \quad y]^\top \in \mathbb{R}^2$ denotes the counter-clock-wise angle from the cartesian coordinate x-axis to the vector $[x \quad y]^\top$.

Note that since the inputs in the agent model defined by (1) are $u_1 = F_A$ and $u_2 = \tau_A$, i.e. v_A and θ_A cannot be applied directly, the terms

$$v_d \triangleq \|Z\|, \ \theta_d \triangleq \angle([-Z_x, -Z_y]^\top) \quad (12)$$

need to be considered as desired set-point values for v_A and θ_A, respectively.

Our objective is to force the motion of agent A such that the differences $|v_A - v_d|$ and $|\theta_A - \theta_d|$ converge to zero. With this objective in mind, similar to [6], [7], [9] and [4], let us define two sliding surfaces, namely one for the linear speed v_A and one for the orientation θ_A, respectively, as

Case 1: $(\|\nabla_{p_A} J(p)\| \neq 0)$

$$\begin{aligned} s_v &= v_A - v_d && (13) \\ s_\theta &= c_\theta(\dot{\theta}_A - \dot{\theta}_d) + (\theta_A - \theta_d), && (14) \end{aligned}$$

Case 2: $(\|\nabla_{p_A} J(p)\| = 0)$

$$\begin{aligned} s_v &= v_A && (15) \\ s_\theta &= c_\theta \dot{\theta}_A + \theta_A, && (16) \end{aligned}$$

where $c_\theta > 0$ is a positive constant, and v_A and θ_A are the actual linear speed and orientation angle, respectively, whereas v_d and θ_d are the desired linear speed and orientation angle as defined in (12). With these definitions, our objective becomes designing the control inputs u_1 and u_2 such that $s_v \to 0$ and $s_\theta \to 0$ in finite time and when they are achieved we will have $v_A \to v_d$ and $\theta_A \to \theta_d$. The existence of the additional term $c_\theta(\dot{\theta}_A - \dot{\theta}_d)$ in (14) comes from the double integration relationship between the terms θ_A and $u_2 = \tau_A$.

It is well known from the sliding mode control theory that if we have the reaching conditions

$$s_v \dot{s}_v \leq -\varepsilon_1 |s_v| \tag{17}$$

$$s_\theta \dot{s}_\theta \leq -\varepsilon_2 |s_\theta| \tag{18}$$

satisfied for some constants $\varepsilon_1, \varepsilon_2 > 0$, then $s_v = 0$ and $s_\theta = 0$ will be achieved in finite time [18], [19].

In order to satisfy (17) we choose the first control input $u_1 = F_A$ as

$$u_1 = -K_1 \mathrm{sgn}(s_v) \tag{19}$$

with which the time derivative of s_v becomes

Case 1: $(\|\nabla_{p_A} J(p)\| \neq 0)$

$$\dot{s}_v = -\frac{K_1}{m_A}\mathrm{sgn}(s_v) + \frac{1}{m_A} f_{v_A} - \dot{v}_d$$

Case 2: $(\|\nabla_{p_A} J(p)\| = 0)$

$$\dot{s}_v = -\frac{K_1}{m_A}\mathrm{sgn}(s_v) + \frac{1}{m_A} f_{v_A}$$

Then, for **Case 1** we have

$$s_v \dot{s}_v \leq -\left(\frac{K_1}{\overline{M}} - \frac{1}{\underline{M}} f_v^+ - \bar{\dot{v}}_d\right)|s_v|$$

whereas for **Case 2** it is the same equation with $\bar{\dot{v}}_d$ omitted. In the equation above, $\bar{\dot{v}}_d$ is a computable upper bound on $\dot{v}_d$ such that $|\dot{v}_d| \leq \bar{\dot{v}}_d$. In other words, we have

$$\begin{aligned} |\dot{v}_d| &\leq \|\dot{Z}\| \\ &\leq \sigma \left\| \frac{d}{dt}(\nabla_{p_A} J(p)) \right\| + \beta \left\| \frac{d}{dt} h(\nabla_{p_A} J(p)) \right\| \\ &\leq \sigma \alpha_1(p) + \beta h_1(p) = \bar{\dot{v}}_d \end{aligned} \tag{20}$$

where $\alpha_1(p)$ and $h_1(p)$ are bounds on the corresponding terms. The existence and properties of $\alpha_1(p)$ depend on the properties of the potential function, which is chosen by the designer. Letting $\nabla_{p_A} J(p) = [J_x \quad J_y]^\top$, for the chosen potential J the value of $\alpha_1(p)$ is computed as

$$\left\| \frac{d}{dt}(\nabla_{p_A} J(p)) \right\| = \|\dot{p}_A - \dot{p}_T\| \leq \|\dot{p}_A\| + \beta_{T_v} \triangleq \alpha_1(p).$$

Similarly, $h_1(p)$ can be computed using the equality $\frac{d}{dt} h(\nabla_{p_A} J(p)) =$

$$\left\{ \begin{array}{l} \left\{ \begin{array}{r} \left(\frac{\pi}{2\bar{\epsilon}}\right) cos\left(\frac{\pi J_x}{2\bar{\epsilon}}\right) \frac{d}{dt}(J_x), |J_x| \leq \bar{\epsilon} \\ 0, |J_x| > \bar{\epsilon} \end{array} \right. \\ \left\{ \begin{array}{r} \left(\frac{\pi}{2\bar{\epsilon}}\right) cos\left(\frac{\pi J_y}{2\bar{\epsilon}}\right) \frac{d}{dt}(J_y), |J_y| \leq \bar{\epsilon} \\ 0, |J_y| > \bar{\epsilon} \end{array} \right. \end{array} \right. . \tag{21}$$

Then by choosing K_1 such that

Case 1: $(\|\nabla_{p_A} J(p)\| \neq 0)$

$$K_1 \geq \frac{\overline{M}}{\underline{M}} \left[\underline{M}\bar{\dot{v}}_d + \underline{M}\varepsilon_1 + f_v^+\right] \tag{22}$$

Case 2: $(\|\nabla_{p_A} J(p)\| = 0)$

$$K_1 \geq \frac{\overline{M}}{\underline{M}} \left[\underline{M}\varepsilon_1 + f_v^+\right] \tag{23}$$

one guarantees that (17) is satisfied and sliding mode occurs (i.e., $s_v = 0$ is satisfied) in finite time.

Similarly, for the second sliding surface in (14) choosing the control input as

$$u_2 = -K_2 \mathrm{sgn}(s_\theta) \tag{24}$$

the time derivative of s_θ becomes

Case 1: $(\|\nabla_{p_A} J(p)\| \neq 0)$

$$\dot{s}_\theta = -c_\theta \frac{K_2}{I_A}\mathrm{sgn}(s_\theta) + \frac{c_\theta}{I_A} f_{w_A} - c_\theta \ddot{\theta}_d + \omega_A - \dot{\theta}_d \tag{25}$$

Case 2: $(\|\nabla_{p_A} J(p)\| = 0)$

$$\dot{s}_\theta = -c_\theta \frac{K_2}{I_A}\mathrm{sgn}(s_\theta) + \frac{c_\theta}{I_A} f_{w_A} + \omega_A \tag{26}$$

Then, for **Case 1** we have

$$s_\theta \dot{s}_\theta \leq -\left(\frac{c_\theta K_2}{\overline{I}} - \frac{c_\theta}{\underline{I}} f_w^+ - c_\theta |\ddot{\theta}_d| - |\dot{\theta}_d| - |\omega_A|\right)|s_\theta| \tag{27}$$

whereas for **Case 2** it is the same equation with $\ddot{\theta}_d$ and $\dot{\theta}_d$ terms omitted.

By choosing K_2 as

Case 1: $(\|\nabla_{p_A} J(p)\| \neq 0)$

$$K_2 \geq \frac{\overline{I}}{c_\theta}\left(\frac{c_\theta}{\underline{I}} f_w^+ + c_\theta \bar{\ddot{\theta}}_d + |\dot{\theta}_d| + |\omega_A| + \varepsilon_2\right), \tag{28}$$

Case 2: $(\|\nabla_{p_A} J(p)\| = 0)$

$$K_2 \geq \frac{\overline{I}}{c_\theta}\left(\frac{c_\theta}{\underline{I}} f_w^+ + |\omega_A| + \varepsilon_2\right), \tag{29}$$

where $\bar{\ddot{\theta}}_d$ is a computable bound (discussed below) such that $|\ddot{\theta}_d| \leq \bar{\ddot{\theta}}_d$, one can guarantee that (18) is satisfied and the second sliding surface $s_\theta = 0$ in (14) will as well be reached in finite time.

In order to be able to compute the value of s_θ one needs the time derivative of θ_d, which is given by

$$\dot{\theta}_d = \frac{\frac{d}{dt}(Z_y) \cdot Z_x - \frac{d}{dt}(Z_x) \cdot Z_y}{(Z_x)^2 + (Z_y)^2}. \tag{30}$$

As mentioned above, the bound ($\bar{\ddot{\theta}}_d \geq |\ddot{\theta}_d|$) on the acceleration of the desired steering angle is needed in order to determine the controller gain K_2. This bound can be defined as

$$|\ddot{\theta}_d| \leq \frac{\|\ddot{Z}\|}{\|Z\|} + 2\left(\frac{\|\dot{Z}\|}{\|Z\|}\right)^2 \triangleq \bar{\ddot{\theta}}_d.$$

In the equation above the value of $\|Z\|$ is known and bounded away from zero since in **Case 1** (only for which it is calculated) we have $\|\nabla_{p_A} J(p)\| \neq 0$ and therefore

$\|Z\| \neq 0$. An upper bound on $\|\dot{Z}\|$ was calculated as $\bar{v}_d$ in equation (20). The term $\|\ddot{Z}\|$ can be calculated as

$$\begin{aligned} \|\ddot{Z}\| &\leq \sigma \left\| \frac{d^2}{dt^2}(\nabla_{p_A} J(p)) \right\| + \beta \left\| \frac{d^2}{dt^2} h(\nabla_{p_A} J(p)) \right\| \\ &\leq \sigma \alpha_2(p) + \beta h_2(p) \end{aligned} \tag{31}$$

where $\alpha_2(p)$ and $h_2(p)$ are bounds on $\left\| \frac{d^2}{dt^2}(\nabla_{p_A} J(p)) \right\|$ and $\left\| \frac{d^2}{dt^2} h(\nabla_{p_A} J(p)) \right\|$, respectively. They can be calculated as

$$\alpha_2(p) = \frac{K_1}{\underline{M}} + \frac{f_v^+}{\underline{M}} + |v_A||w_A| + \beta_{T_a}$$

and using the equation
$\frac{d^2}{dt^2} h(\nabla_{p_A} J(p)) \leq$

$$\left\{ \begin{array}{l} \left\{ \begin{array}{ll} \left(\frac{\pi}{2\bar{\epsilon}}\right) \alpha_2(p) + \left(\frac{\pi j_x}{2\bar{\epsilon}}\right)^2 & , |J_x| \leq \bar{\epsilon} \\ 0 & , |J_x| > \bar{\epsilon} \end{array} \right. \\ \left\{ \begin{array}{ll} \left(\frac{\pi}{2\bar{\epsilon}}\right) \alpha_2(p) + \left(\frac{\pi j_y}{2\bar{\epsilon}}\right)^2 & , |J_y| \leq \bar{\epsilon} \\ 0 & , |J_y| > \bar{\epsilon} \end{array} \right. \end{array} \right. \tag{32}$$

where the accelerations of T and A are bounded such that $\|\ddot{p}_T\| \leq \beta_{Ta}$ and

$$\|\ddot{p}_A\| \leq \frac{K_1}{\underline{M}} + \frac{f_v^+}{\underline{M}} + |v_A||w_A|,$$

respectively.

Once sliding mode occurs on all the surfaces (which happens in finite time), agents start to move according to (7) and from the discussion in the preceding section we know that Problem 1 is solved.

One issue to note about the tracking algorithm discussed here is that after occurrence of sliding mode we reach $v_A = v_d$ but not necessarily $\theta_A = \theta_d$. In fact, after occurrence of sliding mode we have $\theta_A \to \theta_d$ exponentially fast and the speed of convergence depends on the slope of the sliding surface $-\frac{1}{c_\theta}$. Therefore, one needs to choose c_θ as small as possible in order to achieve faster convergence and avoid any instabilities. Note also that decreasing the parameter c_θ will require increasing the controller gain K_{i2}.

We would like to emphasize that the procedure based on the sliding mode control technique presented here will guarantee proper behavior despite the presence of uncertainties in the mass m_A and the inertia I_A of the robots and additive disturbances f_{v_A} and f_{w_A} to the linear and angular speed dynamics which constitute very realistic assumptions.

IV. Simulation Results

In this section we present simulation results to verify the effectiveness of the control scheme proposed in the previous sections.

One issue to note about the algorithm is that the sgn function which is used in the calculation of the control inputs works well in theory. However, in practice it creates numerical problems during simulations. Instead of the sgn function, we used the function $\tanh(\gamma y)$, where γ is a smoothness parameter which determines the slope of the function around $y = 0$ and therefore the similarity between the sgn and $\tanh$ functions. The smoothness parameter in our case is chosen as $\gamma = 20$.

The simulation lasts 50 seconds. The target moves in $\mathbb{R}^2$ with the dynamics

$$\begin{aligned} \dot{x}_T(t) &= 0.05 + 0.1\sin(2t)(m/s), \\ \dot{y}_T(t) &= 1.5\sin(0.5t)(m/s). \end{aligned}$$

From the above equation, the bounds on the velocity and the acceleration of the target are found to be $\beta_{T_v} = 1.5075$ and $\beta_{T_a} = 0.7762$. The parameters in equation (7) are chosen as $\sigma = 0.01$ and $\beta = 1.6$ ($\beta > \beta_{T_v}$). The size of the boundary layer in (8) is chosen as $\bar{\epsilon} = 0.2$.

The actual values of the mass and inertia of agent A are unknown and determined randomly at the beginning of the simulation according to upper and lower bounds $\overline{M} = \overline{I} = 1.2$ and $\underline{M} = \underline{I} = 1.0$. The bounded unmodeled dynamics and disturbances are assumed to be

$$f_v(t) = f_w(t) = 1.2\sin(1.2t)$$

and the corresponding known bounds on them become $f_v^+ = f_w^+ = 1.2$. The slope parameter for the sliding surface s_θ is chosen as $c_\theta = 0.1$ and the sliding mode gains are calculated at every step according to inequalities (22), (23), (28) and (29). The initial position of target T is chosen as $p_T(0) = [3 \quad 3]^\top$. The initial position $p_A(0)$ of agent A is chosen randomly within the square region $[0 \quad 1] \times [0 \quad 1]$.

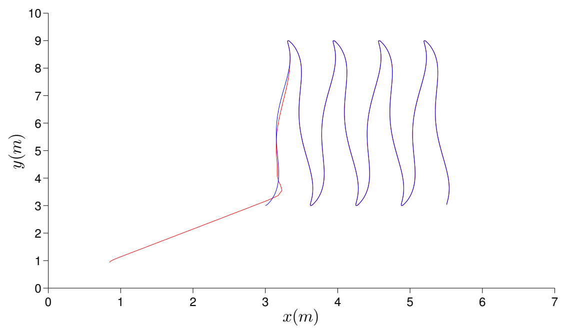

Fig. 2. Paths of Agent A and Target T ($p_T(0) = [3 \quad 3]^\top$).

Figure 2 shows the paths of the target and the agent. It is observed that with random initial positions the agent quickly catches the target and starts to track it with a small error. This implies that $\|p_A(t) - p_T(t)\| \leq \epsilon$ in finite time.

Figure 3 illustrates the satisfaction of equation (2) in Problem 1. In the figure the distance between agent A and target T and its x-axis zoomed version are plotted. It is observed that the distance converges a small region close to zero (the of the error is less than $\bar{\epsilon}$) in finite time ($\approx$ 5 seconds) as expected. The variation of the distance is because of target's movement. When target makes sharp turns the distance increases slightly. This phenomenon can be overcome with the cost of high magnitude gain signals.

The plots in Figures 4 and 5 show the first and the second control inputs u_1 and u_2, respectively, for agent A. High magnitude control signals phenomenon of sliding mode can be observed from the figures. It is seen roughly from the

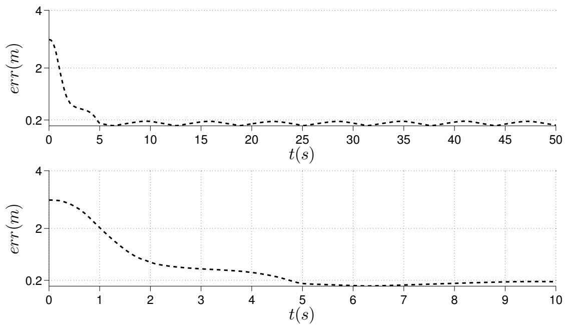

Fig. 3. Distance between the target and agent A.

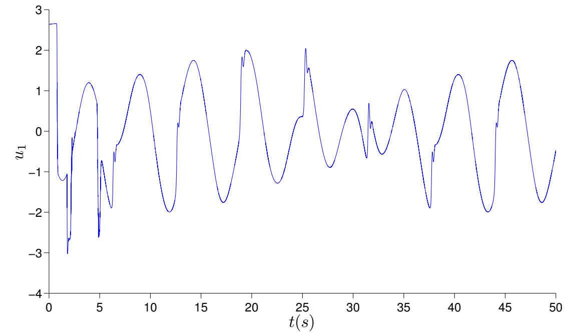

Fig. 4. First control input $u_1 = F_A$.

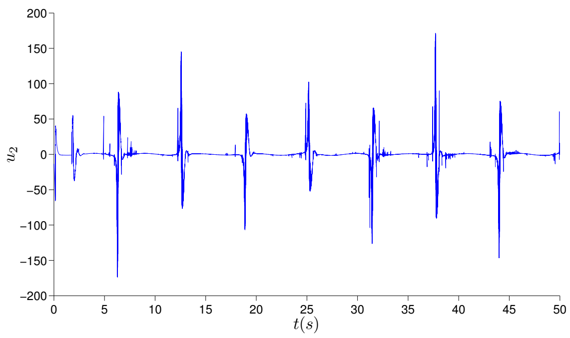

Fig. 5. Second control input $u_2 = \tau_A$.

figures that the control signal gains are bounded as $K_1 \leq 4$ and $K_2 \leq 200$.

V. Concluding Remarks

In this paper, the task of capturing and tracking a maneuvering target has been discussed. In order to realize this control goal, a control scheme based on artificial potential functions and sliding mode techniques has been designed specifically for an agent with non-holonomic unicycle dynamic model, modeling uncertainties and additive disturbances. It has been shown, both theoretically and via simulations, that using the proposed control scheme the agent would intercept the target and track it.

A potential future research direction could be examination of the system performance in the existence of position and distance sensing errors. Issues such as decreasing the magnitude of control signals or achieving tracking under bounded control inputs with pre-defined bounds, which were not considered here, could be also potential subject of future work.

References

[1] V. Gazi and R. Ordonez, "Target tracking using artificial potentials and sliding mode control," *Int. J. Control*, vol. 80, no. 10, pp. 1626–1635, October 2007.

[2] J. Yao, R. Ordonez, and V. Gazi, "Swarm tracking using artificial potentials and sliding mode control," *Journal of Dynamic Systems, Measurement and Control*, vol. 129, no. 5, pp. 749–754, September 2007.

[3] C. M. Saaj, V. Lappas, and V. Gazi, "Spacecraft swarm navigation and control using artificial potential field and sliding mode control," in *IEEE International Conference on Industrial Technology*, Mumbai, India, December 2006, pp. 2646–2652.

[4] V. Gazi, B. Fidan, Y.S. Hanay, and M. İ. Kksal, "Aggregation, foraging, and formation control of swarms with non-holonomic agents using potential functions and sliding mode techniques," *Turkish Journal of Electrical Engineering and Computer Sciences*, vol. 15, no. 2, pp. 149–168, July 2007.

[5] J. Guldner and V. I. Utkin, "Sliding mode control for an obstacle avoidance strategy based on an harmonic potential field," in *Proc. Conf. Decision Contr.*, San Antonio, Texas, December 1993, pp. 424–429.

[6] J. Guldner and V. I. Utkin, "Stabilization on nonholonomic mobile robots using Lyapunov functions for navigation and sliding mode control," in *Proc. Conf. Decision Contr.*, Lake Buena Vista, Florida, December 1994, pp. 2967–2972.

[7] J. Guldner, V. I. Utkin, H. Hashimoto, and F. Harashima, "Tracking gradients of artificial potential fields with non-holonomic mobile robots," in *Proc. American Control Conf.*, Seattle, Washington, June 1995, pp. 2803–2804.

[8] J. Guldner and V. I. Utkin, "Sliding mode control for gradient tracking and robot navigation using artificial potential fields," *IEEE Trans. on Robotics and Automation*, vol. 11, no. 2, pp. 247–254, April 1995.

[9] J. Guldner and V. I. Utkin, "Tracking the gradient of artificial potential fields: Sliding mode control for mobile robots," *Int. J. Control*, vol. 63, no. 3, pp. 417–432, 1996.

[10] E. Rimon and D. E. Koditschek, "Exact robot navigation using artificial potential functions," *IEEE Trans. on Robotics and Automation*, vol. 8, no. 5, pp. 501–518, October 1992.

[11] O. Khatib, "Real-time obstacle avoidance for manipulators and mobile robots," *The International Journal of Robotics Research*, vol. 5, no. 1, pp. 90–98, 1986.

[12] H. Yamaguchi, "A cooperative hunting behavior by mobile-robot troops," *The International Journal of Robotics Research*, vol. 18, no. 8, pp. 931–940, September 1999.

[13] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: Algorithms and theory," *IEEE Trans. on Automatic Control*, vol. 51, no. 3, pp. 401–420, March 2006.

[14] M. Egerstedt and X. Hu, "Formation constrained multi-agent control," *IEEE Trans. on Robotics and Automation*, vol. 17, no. 6, pp. 947–951, December 2001.

[15] J. H. Reif and H. Wang, "Social potential fields: A distributed behavioral control for autonomous robots," *Robotics and Autonomous Systems*, vol. 27, pp. 171–194, 1999.

[16] V. Gazi and K. M. Passino, "A class of attraction/repulsion functions for stable swarm aggregations," *Int. J. Control*, vol. 77, no. 18, pp. 1567–1579, December 2004.

[17] V. Gazi, "Swarm aggregations using artificial potentials and sliding mode control," *IEEE Trans. on Robotics*, vol. 21, no. 6, pp. 1208–1214, December 2005.

[18] V. I. Utkin, *Sliding Modes in Control and Optimization*, Springer Verlag, Berlin, Heidelberg, 1992.

[19] V. I. Utkin, J. Guldner, and J. Shi, *Sliding Mode Control in Electromechanical Systems*, CRC Press, Boca Raton, FL, 1999.